From linear to hydrodynamic fluctuations

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1. Introduction

The strong coupling between longitudinal and Goldstone modes in Bose gases leads to known IR divergences in perturbation theory [1]. As a non-perturbative formalism the Functional renormalisation group (FRG) can overcome these divergences [2], however IR cancellations are still not respected due to the truncation of the action.

Popov developed an hydrodynamic effective theory to describe the low-momentum regime of Bose gases [3]. He introduced an Amplitude-Phase (AP) representation for the boson fields which ease the correct treatment of the phase fluctuations. Popov's ideas led to the concept of quasi-condensate [4], which is particularly useful in the study of low-dimensional systems.

Following Popov's ideas, we implement scale-dependent fields in the FRG that interpolates between a Cartesian representation for high-momentum and an AP one for low-momentum. We study $O(2)$ models in two and three dimensions in order to test our approach.

2. Effective action

The flow of the effective action Γ is dictated by [5]

$$\partial_k\Gamma + \dot{\Phi}\cdot\frac{\delta\Gamma}{\delta\Phi} = \mathrm{tr}\left[\left(\frac{1}{2}\partial_k\mathbf{R} + \dot{\Phi}^{(1)}\mathbf{R}\right)\left(\Gamma^{(2)} - \mathbf{R}\right)^{-1}\right]$$

where $\dot{\Phi} = \partial_k\Phi$ and $\dot{\Phi}^{(1)} = \delta\dot{\Phi}/\delta\Phi$. We consider the O(2) Ansatz

$$\Gamma = \frac{-1}{T}\int_{\mathbf{x}}\left[\frac{Z_m}{2m}\nabla\phi^\dagger\nabla\phi + \frac{Y_m}{8m}(\nabla\rho)^2 + U(\rho,\mu)\right], \qquad (*)$$

where $\rho = \phi^\dagger\phi$. We truncate the potential as [6]

$$U(\rho,\mu) = u_0 + u_1(\rho-\rho_0) + \frac{u_2}{2}(\rho-\rho_0)^2 - n_0(\mu-\mu_0) - n_1(\mu-\mu_0)(\rho-\rho_0) - \frac{n_2}{2}(\mu-\mu_0)(\rho-\rho_0)^2,$$

where μ_0 is the physical chemical potential. At $k=0$ we obtain the physical boson density n_0 and superfluid density $\rho_s = Z_m\rho_0$.

3. Interpolating fields

We use the k-dependent fields $\Phi = (\sigma,\vartheta)$ defined by [7]

$$\phi = (\sigma + b_k)e^{i\vartheta/b_k} - (b_k - \sqrt{\rho_0}), \qquad b_k \in [\sqrt{\rho_0},\infty) \qquad (**)$$

The fields change representation as b_k varies with k. In the limits ϕ take the forms

$$\phi = \begin{cases}(\sqrt{\rho_0}+\sigma) + i\vartheta & : b_k \to \infty \quad \textbf{(Cartesian)},\\ (\sqrt{\rho_0}+\sigma)e^{i\vartheta/\sqrt{\rho_0}} & : b_k = \phi_0 \quad \textbf{(AP)}.\end{cases}$$

ρ_0 has different meanings in each limit. From the long-distance behaviour of the correlation function $G_n(\mathbf{x}) = \langle\phi^\dagger(\mathbf{x})\phi^\dagger(0)\rangle$

$$\lim_{|\mathbf{x}|\to\infty} G_n(\mathbf{x}) = \begin{cases}\rho_c & : \textbf{(Cart.)} \qquad \rho_0 = \rho_c,\\ \rho_q e^{\frac{1}{2\rho_q}\langle(\vartheta(\mathbf{x})-\vartheta(0))^2\rangle} & : \textbf{(AP)} \qquad \rho_0 = \rho_q,\end{cases}$$

with ρ_c the condensate density and ρ_q the quasi-condensate density. In condensed systems, such as in three dimensions, both ρ_c and ρ_q are finite and ρ_c is the order parameter. In superfluid systems without a broken symmetry, such as in two dimensions at finite temperature, ρ_q is finite while $\rho_c = 0$.

4. Evolution in the broken phase

By inserting the interpolating fields $(**)$ into the ansatz for Γ $(*)$ we obtain the parametrisation

$$\Gamma = -\frac{1}{T}\int_{\mathbf{x}}\left[\frac{Z_\vartheta}{2m}A_k^2(\sigma)(\nabla\vartheta)^2 + \frac{Z_\sigma(\vartheta)}{2m}(\nabla\sigma)^2 + \frac{Y_m}{2m}C_k(\sigma,\vartheta,\nabla\sigma,\nabla\vartheta) + U(\rho,\mu)\right],$$

where $Z_\vartheta = Z_m$ and $Z_\sigma = Z_\vartheta + \rho_0 Y_m$ at $\rho = \rho_0$. See specific details on Ref. [8].

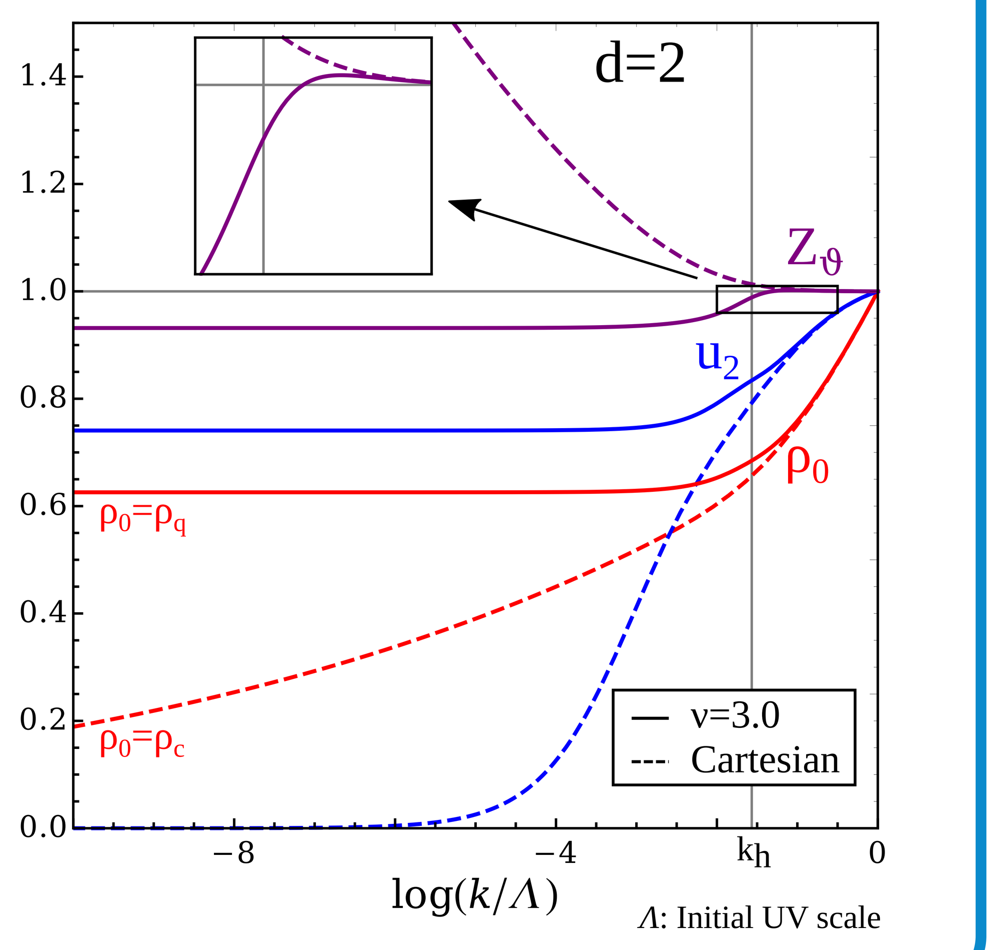
The transition should be made around the "healing scale" k_h defined by

$$w_{k_h} = \frac{Z_\sigma k_h^2/2m}{2u_2\rho_0} = 1.$$

The Cartesian representation should be used for $k \gg k_h$, and the AP representation for $k \ll k_h$ where Goldstone fluctuations dominate. We choose

$$b_k = \phi_0\left[1 + w_k^\nu\right].$$

where ν controls how fast the transition is made.

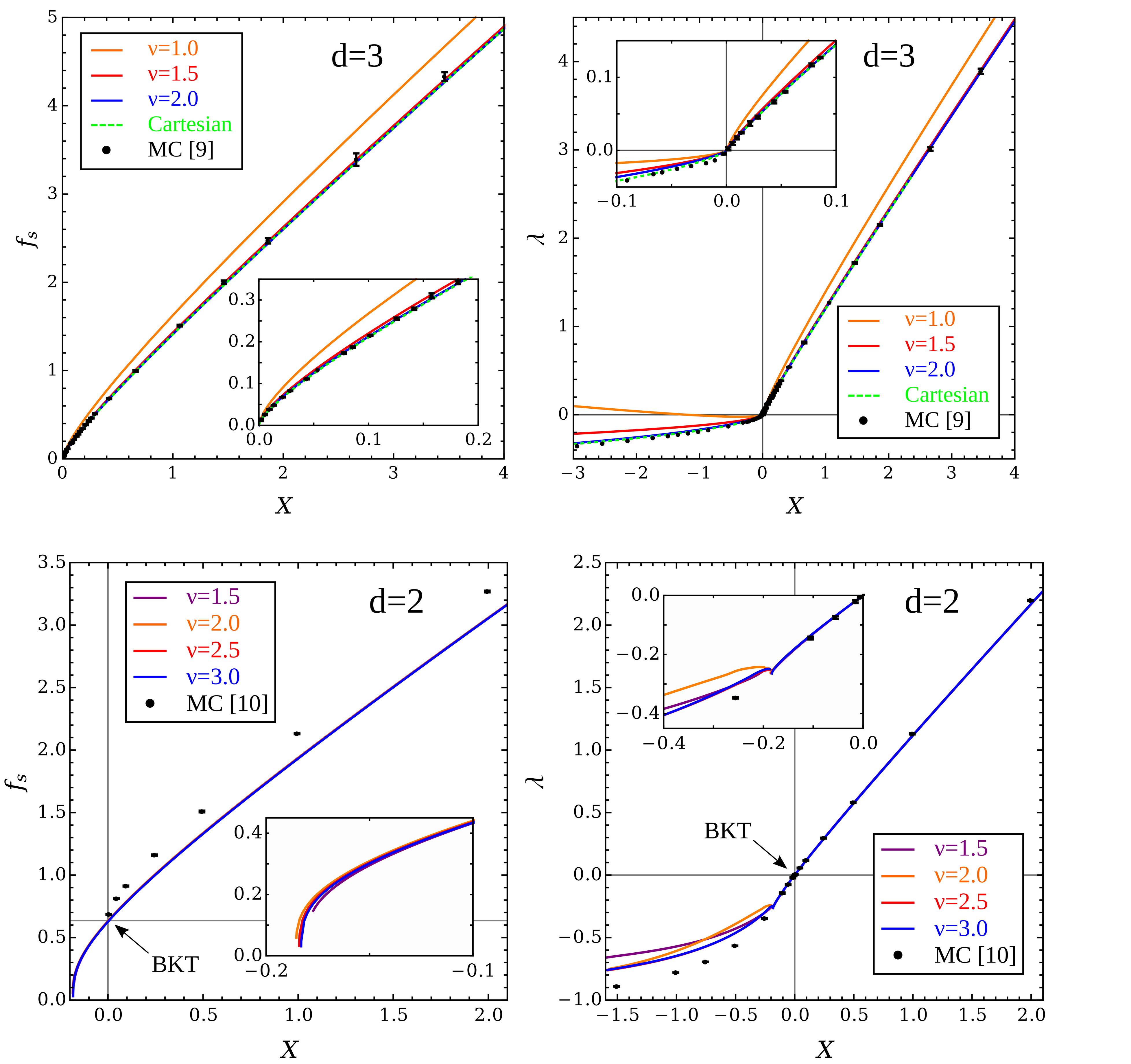


5. Results and conclusions

We present results for two and three dimensions. We study the dimensionless functions

$$f_s = \rho_s/(m^dT^2g^{d-2})^{\frac{1}{4-d}}, \qquad \lambda = (n_0 - n_c)/(m^dT^2g^2)^{\frac{1}{4-d}}, \qquad n_c\text{: critical boson density}$$

which are universal functions of the dimensionless control parameter $X = (\mu_0-\mu_c)/(m^dT^2g^2)^{\frac{1}{4-d}}$.



- The results converge for $\nu \geq 2.5$. In $d=3$ we obtain a good agreement with the simulations.
- We obtain a stable superfluid phase in $d=2$ and a reasonable good agreement with the simulations. This is achieved by working in terms of a quasi-condensate at low scales.
- The deviations in $d=2$ are expected since vortex effects were not included.

References

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