Counterflow of impurities in harmonically confined optical lattices

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I. Background

The experimental realisation of **ultracold atomic mixtures** [1] has revitalised the interest in studying **impurities** immersed in a **quantum medium** [2,3]. In this direction, atomic impurities confined in **optical lattices** [3] have emerged as a rich platform for studying polaron physics. For example, impurities interacting with **bosonic baths** appear to display intriguing features across the **superfluid-to-Mott insulator** (SF-MI) transition [5-7].

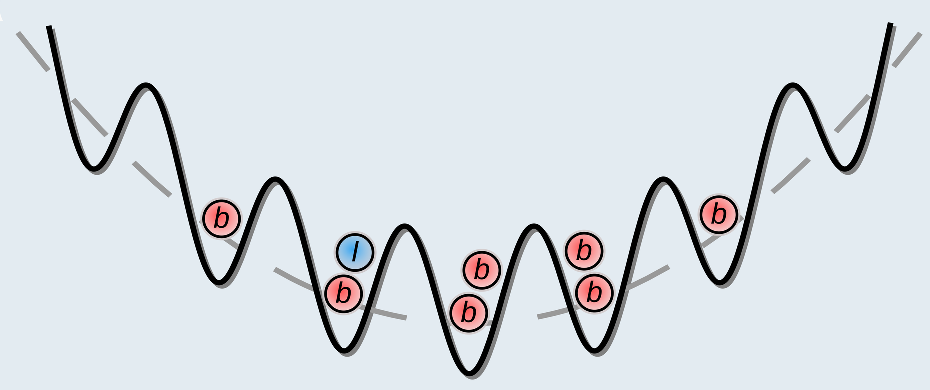


Illustration of the system under consideration. A one-dimensional harmonically confined lattice is loaded with a bath of bosons (red circles) and one mobile impurity (blue hatched circle).

In this work, we study a mobile **impurity** interacting with a **bosonic bath** and immersed in a **one-dimensional harmonically confined optical lattice**. We reveal that the impurity can form a **counterflow** state with the bath for a selected range of interactions.

II. Model

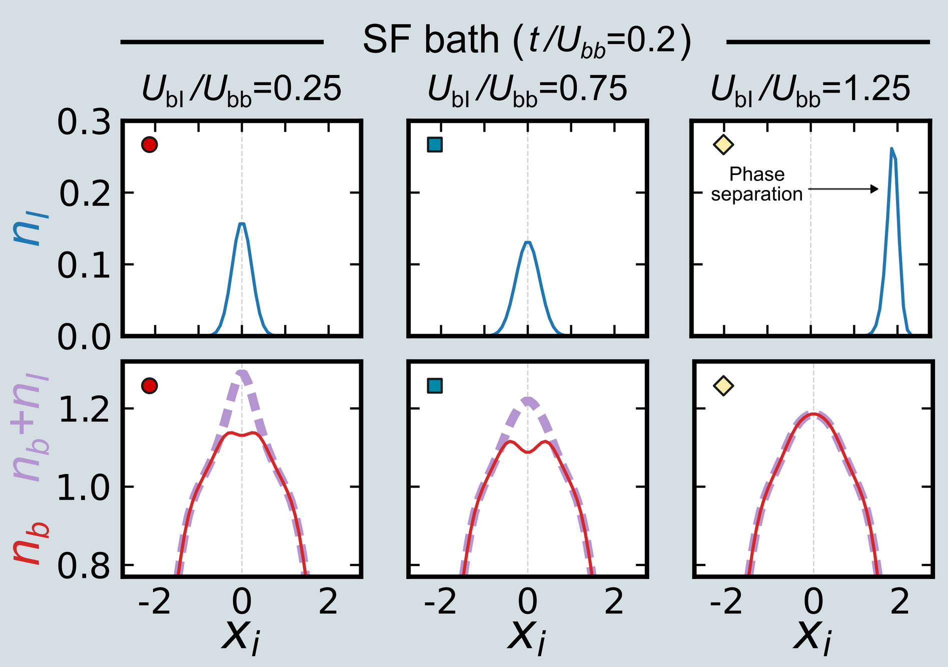
We consider a **two-component Bose-Hubbard Hamiltonian**

$$\hat{H} = -t\sum_{\sigma=b,I}\sum_{i=1}^{M}(\hat{a}^\dagger_{i,\sigma}\hat{a}_{i+1,\sigma} + \text{h.c}) + V_{\text{ho}}\sum_{\sigma=b,I}\sum_{i=1}^{M}(i-i_0)^2\hat{n}_{i,\sigma}$$
$$+\frac{U_{bb}}{2}\sum_{\sigma=b,I}\sum_{i=1}^{M}\hat{n}_{i,\sigma}(\hat{n}_{i,\sigma}-1) + U_{bI}\sum_{i=1}^{M}\hat{n}_{i,b}\hat{n}_{i,I},$$

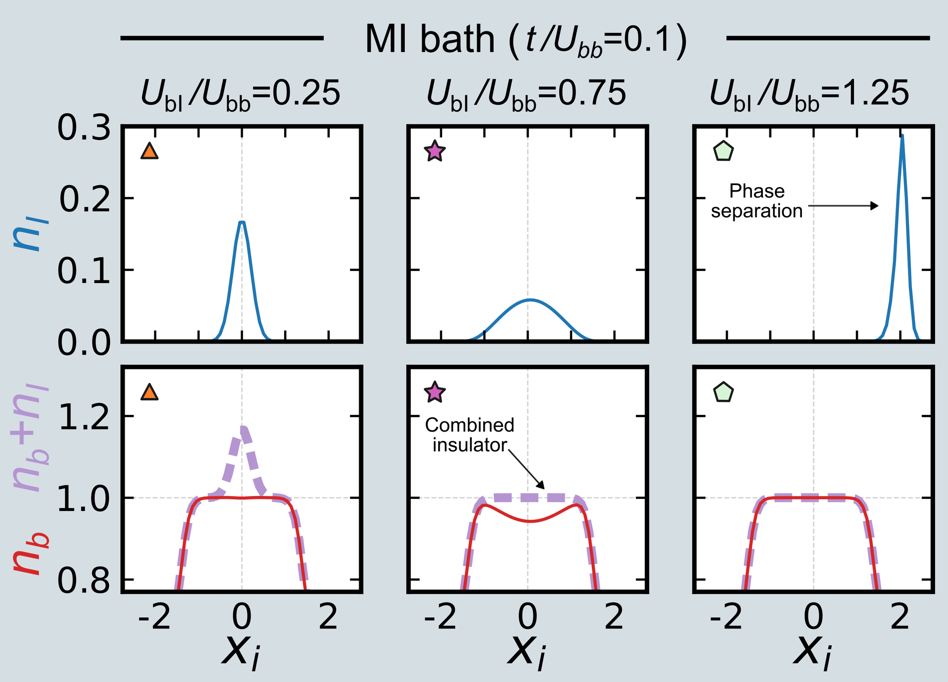
where b and I denote the bath's bosons and impurities, respectively, M is the number of sites, and $i_0=(M+1)/2$. We perform **DMRG** simulations [8,9] for M=60 and N_b=40 bosons in the bath.

III. Density profiles

We study the **density profiles** $n_\sigma(i) = \langle \hat{n}_{i,\sigma}\rangle$ of each species.



In an **SF bath**, the impurity localises at the centre of the trap (**miscible**) for $U_{bI}<U_{bb}$, while it **phase-separates** for $U_{bI}>U_{bb}$.

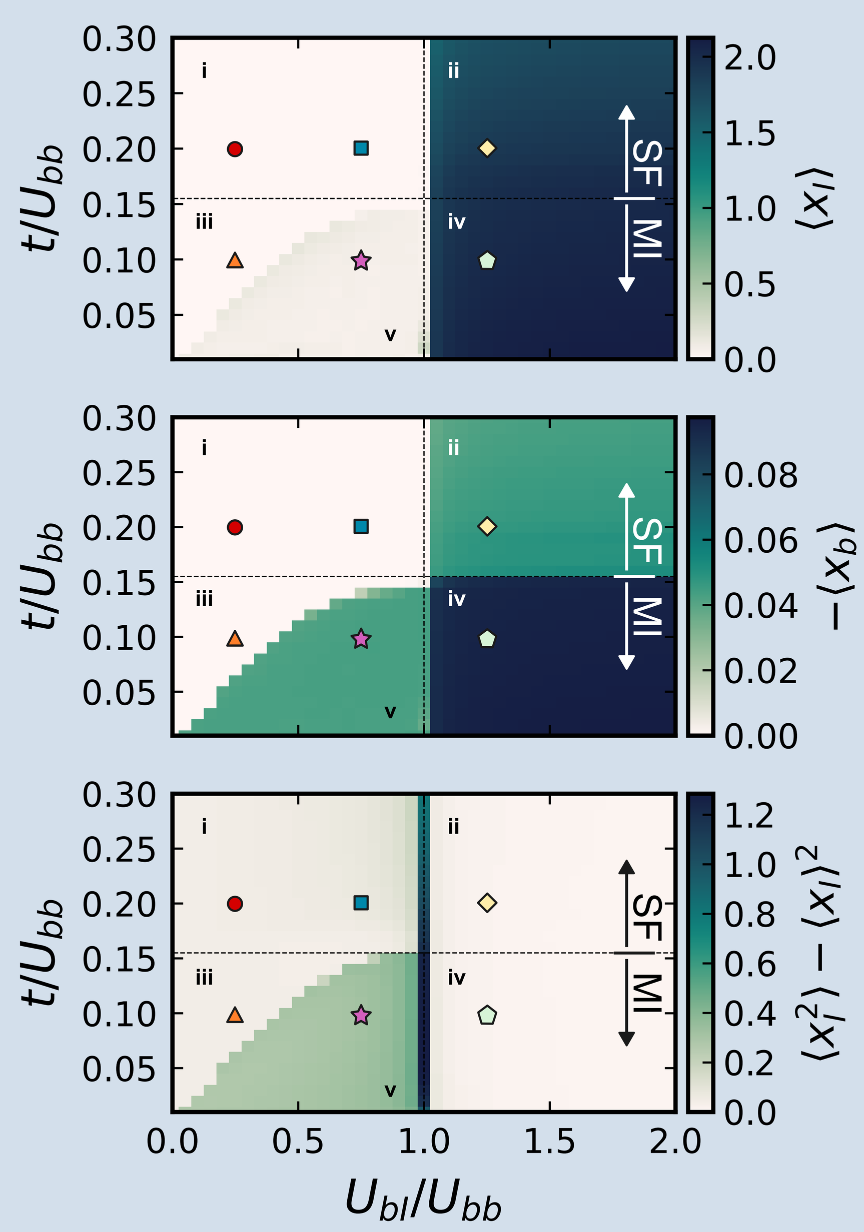


In an **MI bath**, the impurity also localises at the centre of the trap (**miscible**) for small U_{bI} and **phase-separates** for $U_{bI}>U_{bb}$. However, for intermediate U_{bI}, the impurity and bath form a **combined insulator**.

$x_i=(i-i_0)/\xi, \qquad \xi=\sqrt{t/V_{\text{ho}}}.$

IV. Phase diagram

To compute a phase diagram, we examine the **average position** of each species and the size of the **impurity cloud,**



$$\langle x_\sigma\rangle = \frac{1}{N_\sigma}\sum_{i=1}^{M}x_i n_\sigma(i), \qquad \sigma=b,I$$
$$\langle x_I^2\rangle = \sum_{i=1}^{M}x_i^2 n_I(i).$$

The system shows **well-defined phases**.

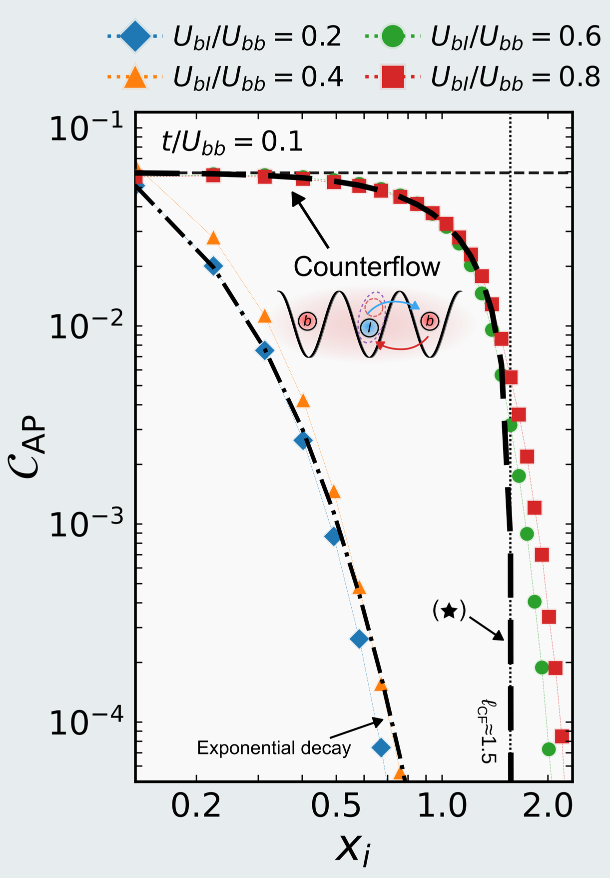
- i: SF + miscible
- ii: SF + phase separation
- iii: MI + miscible
- iv: MI + phase separation
- v: Combined insulator (**counterflow**)

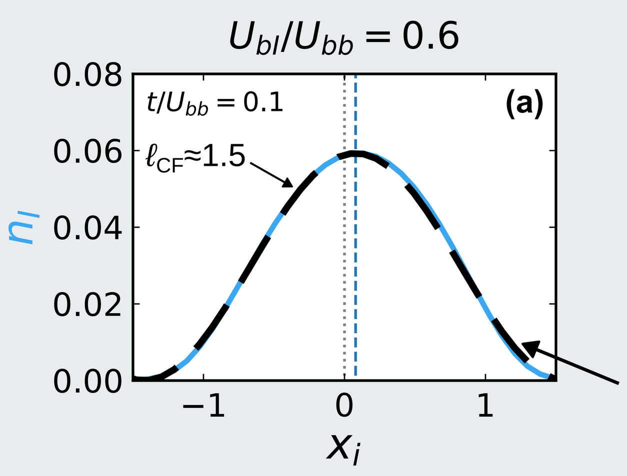
V. Counterflow phase

Region v corresponds to a **counterflow** [10] phase with **anti-pair order** [11],

$$\mathcal{C}_{\text{AP}} = \langle \hat{a}_{b,0}\hat{a}^\dagger_{I,0}\hat{a}^\dagger_{b,i}\hat{a}_{I,i}\rangle.$$

Supercounterflows were **realised experimentally** very recently with **binary Mott insulators** [12]. However, our results show that **counterflows** appear for a **large population imbalance**.





Importantly, the impurity forms this **correlated** state with almost the **whole bath**. Additionally, the **impurity** shows the profile of a **free particle** in a **square well**,

$$n_I^{(\text{CF})}(x_I) = n_i^{(0)}\cos^2(\pi(x_i-\langle x_I\rangle)/\ell_{\text{CF}}).$$

By using an **impurity-hole** toy model, we can obtain a simple analytical expression for the **correlator**,

$$\mathcal{C}_{\text{AP}}(x_I) = n_i^{(0)}\cos\left(\pi(x_i-\langle x_I\rangle)/\ell_{\text{CF}}\right), \qquad (\star)$$

which correctly describes its behaviour in the counterflow regime.

VI. Conclusions and outlook

We have found that an **impurity** can form a correlated **counterflow** state with a **bosonic bath** in one-dimensional optical lattices. This phase shows peculiar features, such as the impurity behaving like a free particle in a square well. Future work will include the consideration of **multiple impurities** and the examination of **dynamics**.

References

[1] C. Baroni, G. Lamporesi, M. Zaccanti, Nat. Rev. Phys. **6**, 736 (2024).
[2] F. Grusdt *et al.*, Rep. Prog. Phys. **88**, 066401 (2025).
[3] P. Massignan *et al.*, arXiv:2501.09618.
[4] I. Bloch, Nat. Phys. **1**, 23 (2005).
[5] V. E. Colussi *et al.*, PRL **130**, 173002 (2023).
[6] R. Alhyder *et al.*, SciPost Phys. **19**, 002 (2025).
[7] T. Hartweg, T. Gupta, G. Pupillo, PRB **112**, L220201 (2025).
[8] S. R. White, PRL **69**, 2863 (1992).
[9] U. Schollwöck, Rev. Mod. Phys. **77**, 259 (2005).
[10] A. B. Kuklov, B. V. Svistunov, PRL **90**, 100401 (2003).
[11] A. Hu *et al.*, PRA **80**, 023619 (2009).
[12] Y.-G. Zheng *et al.*, Nat. Phys. **21**, 208 (2025).

Acknowledgments

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