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Functional renormalization group for cold atom mixtures

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F. Isaule, I. Morera, A. Polls and B. Juliá-Díaz,
PRA **103**, 013318 (2020)

Outline

1. Functional renormalization for cold atom systems
2. Repulsive Bose-Bose mixtures
3. The Bose polaron
4. Conclusions

Example: Weakly interacting Bose gas

- In a field theory formulation, a weakly interacting Bose gas is described by the microscopic action

$$\mathcal{S} = \int_x \left[\varphi^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \varphi + \frac{g}{2} (\varphi^\dagger \varphi)^2 \right]$$

g: repulsive contact potential

- It defines the grand-canonical partition function

$$\mathcal{Z}[\varphi] = \int D(\varphi, \varphi^\dagger) e^{-\mathcal{S}[\varphi]} \quad \rightarrow \quad \begin{aligned} \Omega &= -T \ln \mathcal{Z} \\ d\Omega &= -P dV - S dT - N d\mu \end{aligned}$$

Ω: grand-canonical potential

Effective action

- To obtain *Z* we need to integrate the different paths

$$\mathcal{Z}[\varphi] = \int D(\varphi, \varphi^\dagger) e^{-\mathcal{S}[\varphi]}$$

- Approximations: mean-field, Gaussian
L. Salasnich and F. Toigo, Phys. Rep. **640**, 1 (2016)

- An alternative is to work in terms of an **effective action** Γ that already contains the effect of fluctuations

$$\Gamma[\phi] = -\ln \mathcal{Z}_J[\phi] + \int_x J \cdot \phi \quad \rightarrow \quad \Omega = -T\,\Gamma[\phi_0]$$

$\phi(x) = \langle \varphi(x) \rangle$

J: source fields

- The are different ways to compute Γ. For example, in a perturbative expansion:

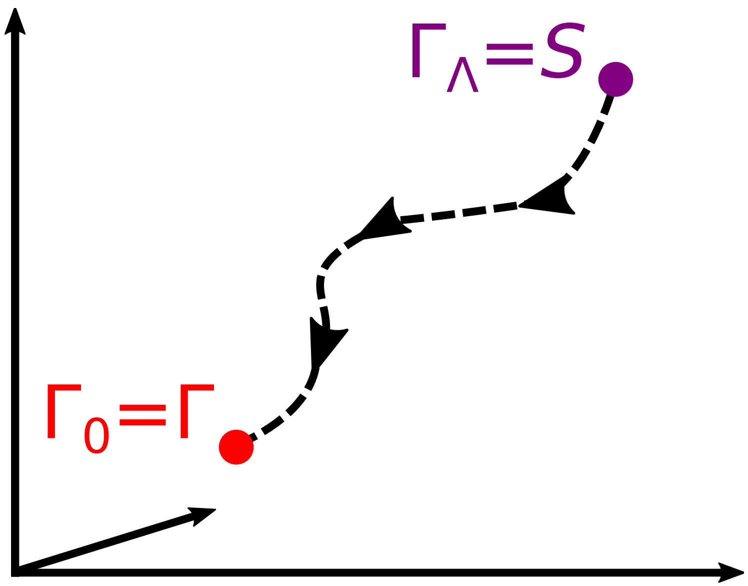
$$\Gamma[\phi] = \mathcal{S}[\phi] + \frac{1}{2} \mathrm{tr} \ln \left(\mathcal{S}^{(2)}[\phi] \right) + \ldots$$

Functional renormalization group (FRG)

- The FRG is a modern **non perturbative** formulation of Wilson's RG.
- A regulator function R_k is added to the theory. It **suppresses all fluctuations for $q<k$**.
- We work in terms of a k-dependent effective action $\boldsymbol{\Gamma_k}$. We follow its flow with a RG equation

$$\boxed{\partial_k \Gamma_k = \frac{1}{2}\mathrm{tr}\left[\partial_k R_k (\Gamma^{(2)} + R_k)^{-1}\right]}$$

C. Wetterich, Phys. Lett. B **301**, 90 (1993)



Functional renormalization group (FRG)

- The FRG is used in a variety of fields: high-energy physics, condensed matter, statistical physics, etc.
- It is particularly useful to study **strongly correlated systems** and critical phenomena.

Recent comprehensive review: N. Dupuis *et al.*, arXiv:2006.04853

- In has been used in different cold atom problems:
  - One-component Bose gases
    S. Floerchinger and C. Wetterich, PRA **77**, 053603 (2008). PRA **79**, 013601 (2009).
  - Fermi gases: BCS-BEC crossover
    S. Floerchinger *et al.*, PRA **81**, 063619 (2010). I. Boettcher *et al.*, PRA **89**, 053630 (2014).
  - Bose gases in optical lattices
    A. Rançon and N. Dupuis, PRA **85**, 063607 (2012), PRA **86**, 043624 (2012).
  - Few bosons: Efimov physics
    S. Floerchinger *et al.*, Few-Body Syst. **51**, 153 (2011). R. Schmidt and S. Moroz, PRA **81**, 052709 (2010).
  - Fermi polaron
    R. Schmidt and T. Enss, PRA **83**, 063620 (2011).

Example: Weakly interacting Bose gas

- The microscopic action:

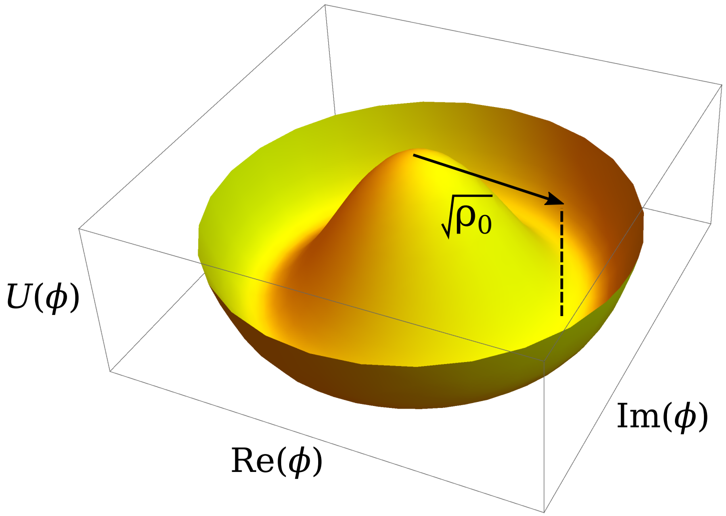
$$\mathcal{S} = \int_x \left[\varphi^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \varphi + \frac{g}{2} (\varphi^\dagger \varphi)^2 \right]$$

- We propose an ansatz based on a **derivative expansion**:

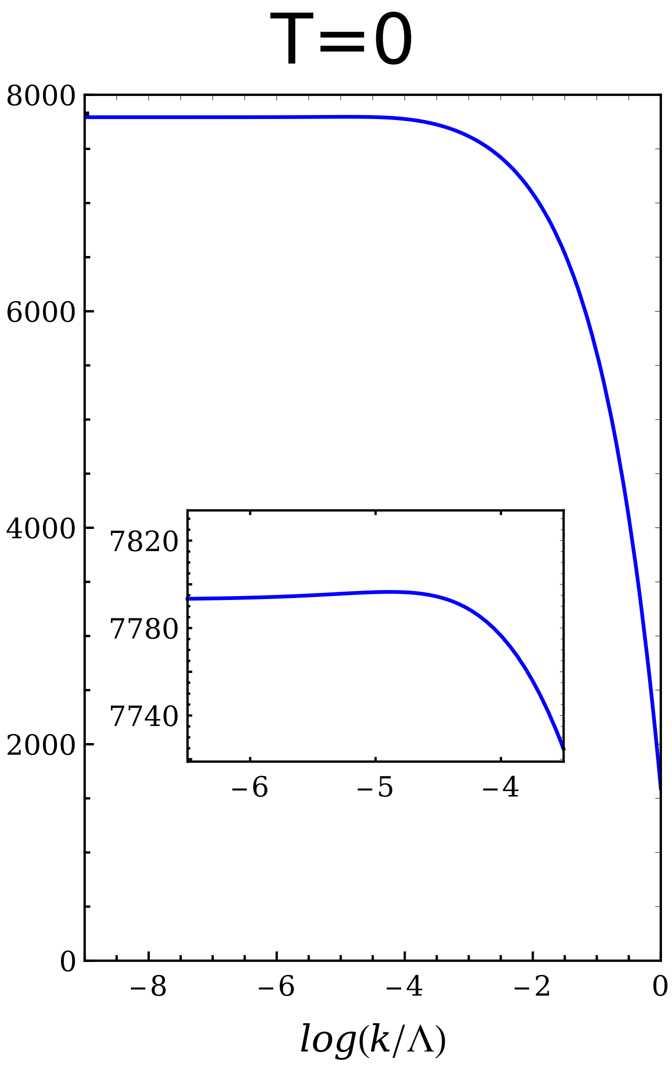
$$\Gamma_k[\phi] = \int_x \left[\phi^\dagger \left(S\partial_\tau - \frac{Z}{2m}\nabla^2 - V\partial_\tau^2 \right) \phi + U(\rho) \right]$$

$$U(\rho) = -P + u(\rho - \rho_0) + \frac{\lambda}{2}(\rho - \rho_0)^2, \qquad \rho = \phi^\dagger \phi$$

- Couplings flow with *k*
- ρ_0 : condensate density
- Physical inputs: μ, a, T



RG flows: Weakly interacting Bose gas (d=3)



ρ_0: condensate density

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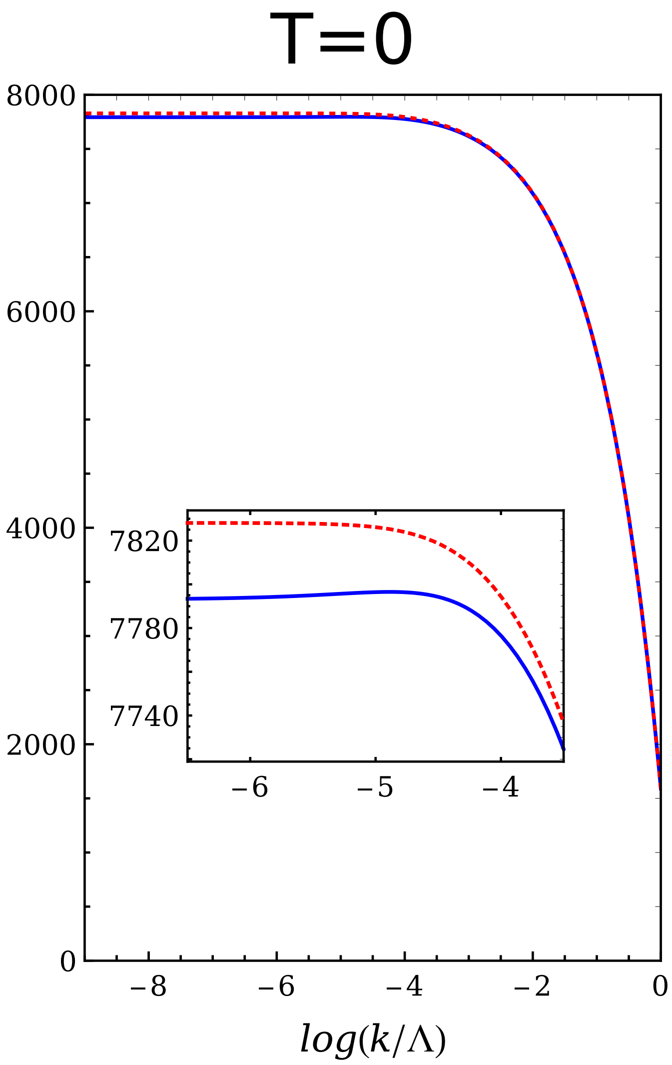

ρ_0: condensate density

n: boson density

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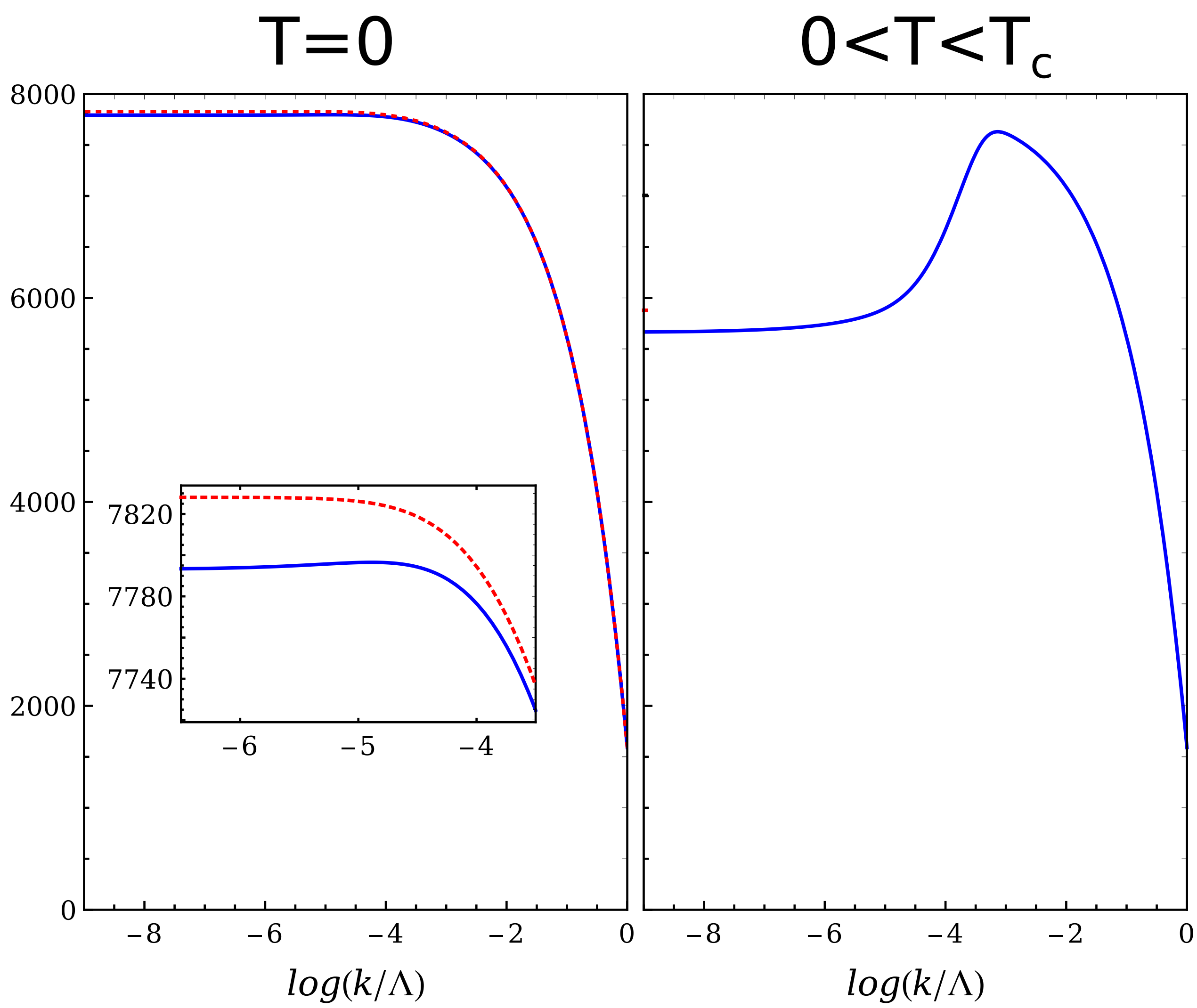

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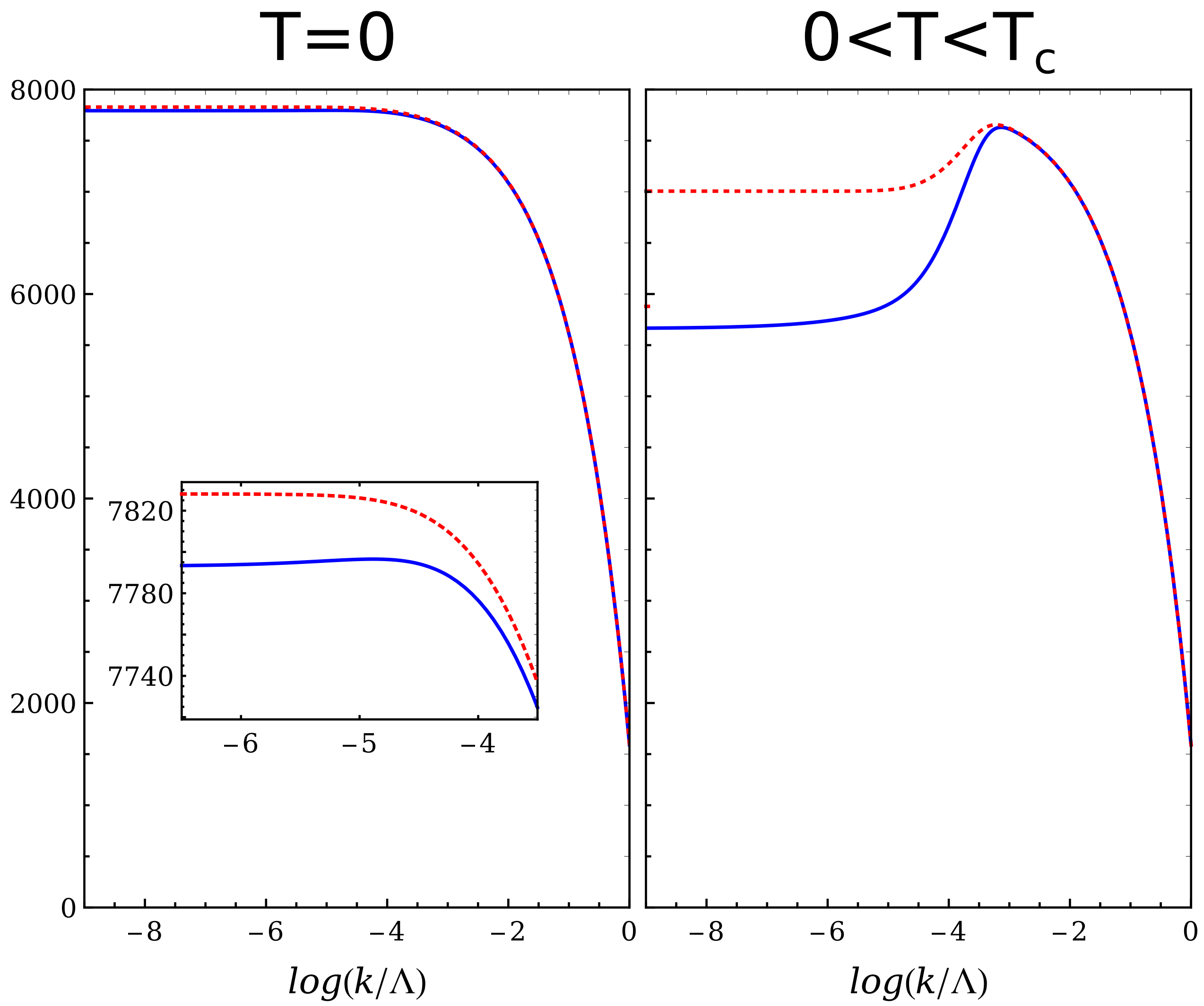

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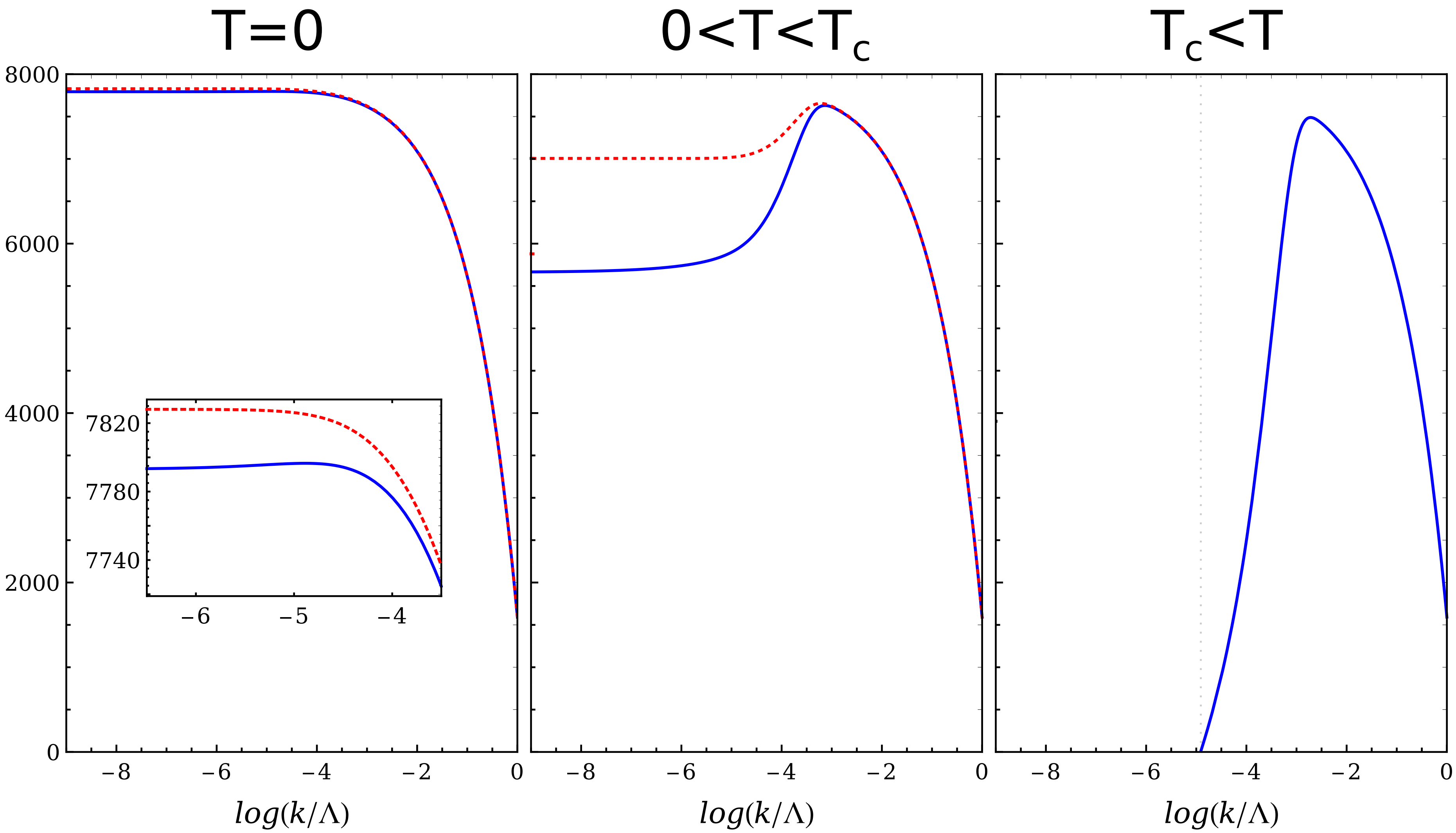

ρ_0: condensate density

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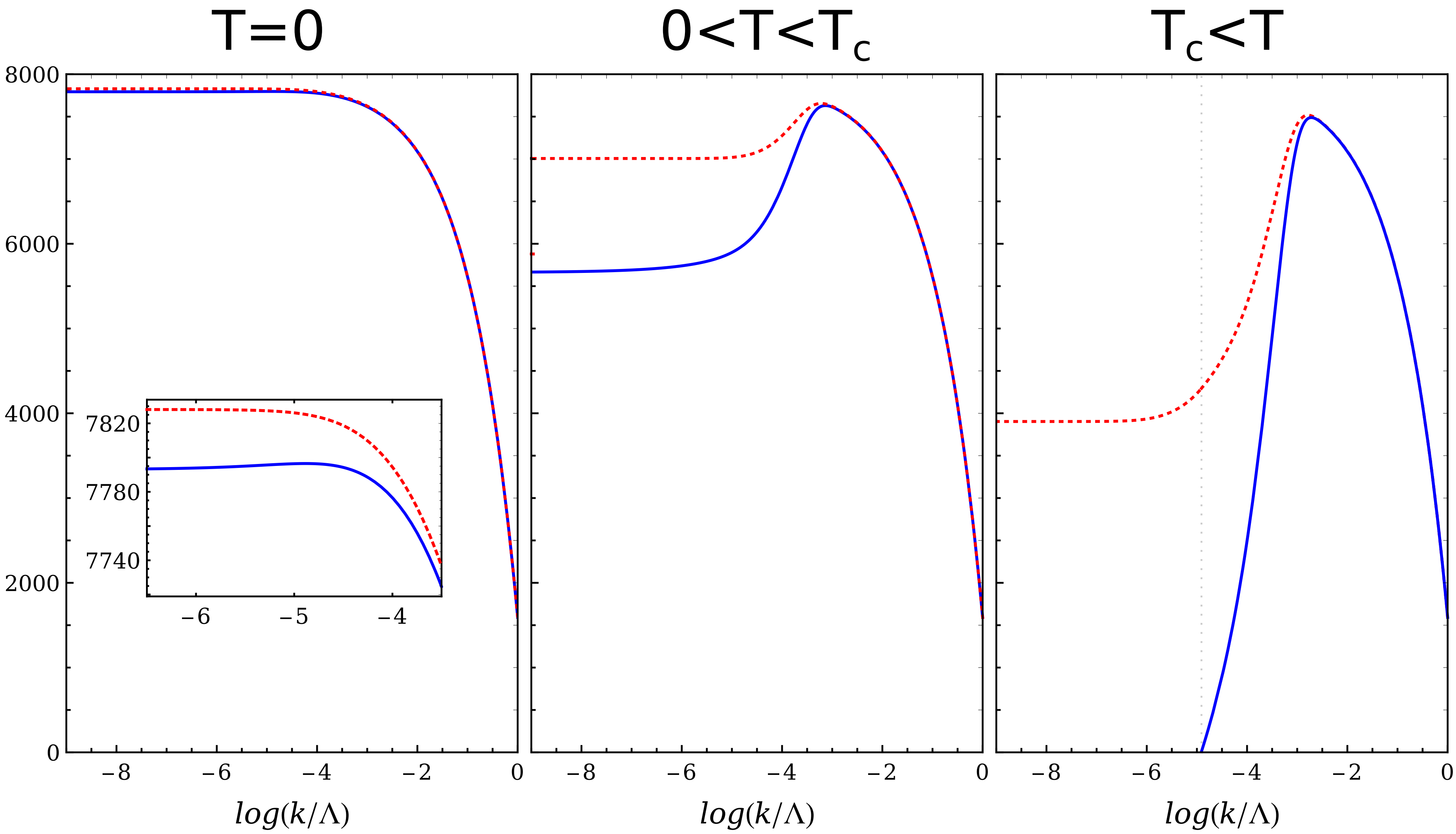
RG flows: Weakly interacting Bose gas (d=3)

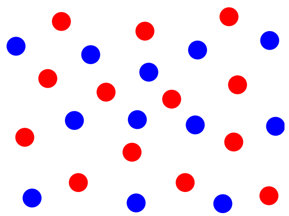


ρ_0: condensate density

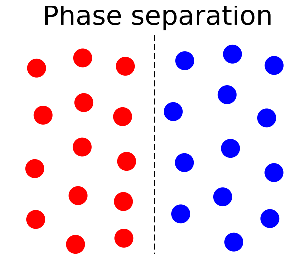
n: boson density

Bose-Bose mixtures

- Gases with two species of bosons have attracted significant attention in recent years



- The interplay between the two component of the gas leads to rich physics:
  - Spin drag
  - Phase separation (repulsive interspecies potential)
  - Self-bound droplets (attractive interspecies potential)
    D. Petrov, PRL **115**, 155302 (2015)



- We study balanced and repulsive Bose-Bose mixtures with the FRG

FRG for repulsive Bose-Bose mixtures

- We consider a balanced Bose-Bose mixture at T=0

$$m_A = m_B, \quad \mu_A = \mu_B, \quad a_{AA} = a_{BB}$$

- We propose the following ansatz:

$$\Gamma_k[\phi] = \int_x \left[\sum_{a=A,B} \psi_a^\dagger \left(S\partial_\tau - \frac{Z}{2m}\nabla^2 - V\partial_\tau^2 \right) \psi_a + U(\rho_A, \rho_B) \right]$$

$$U(\rho_A, \rho_B) = -P + \frac{\lambda}{2}(\rho_A - \rho_0)^2 + \frac{\lambda}{2}(\rho_B - \rho_0)^2 + \lambda_{AB}(\rho_A - \rho_0)(\rho_B - \rho_0)$$

$$\rho_a = \psi_a^\dagger \psi_a$$

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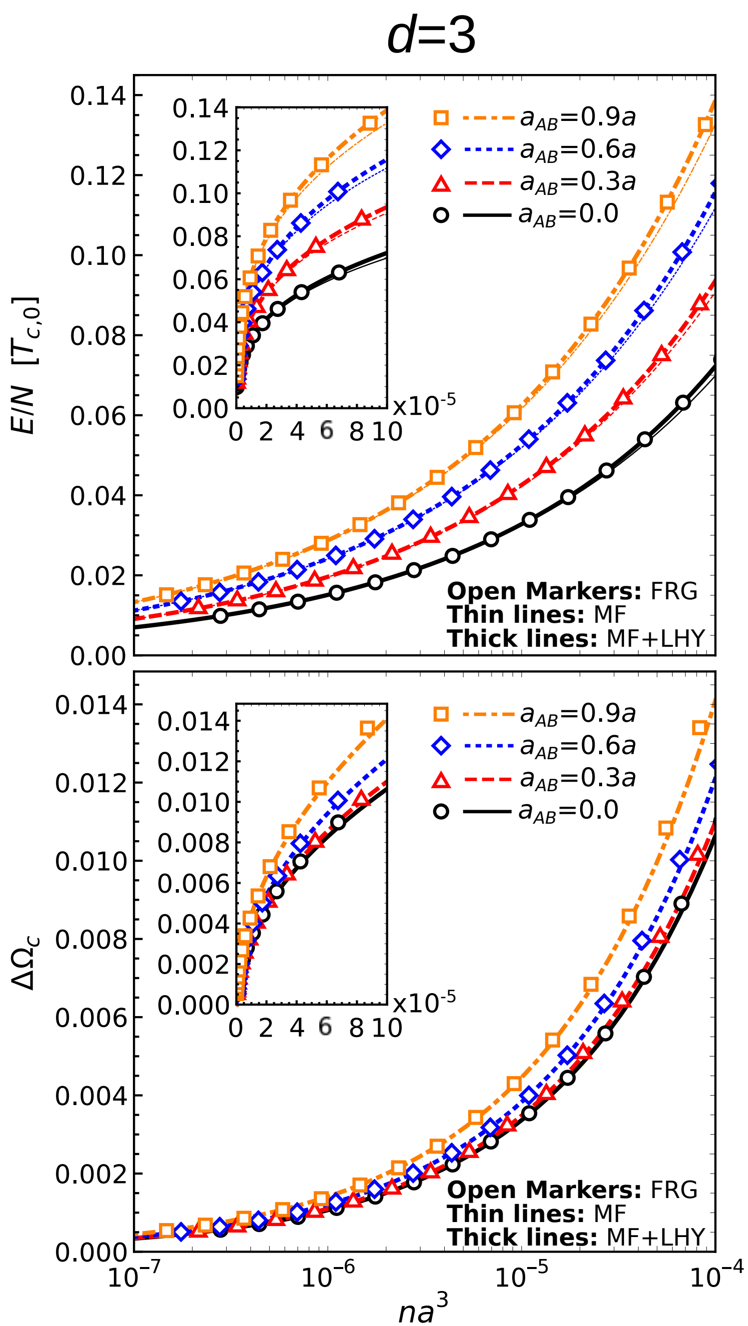
$$\rho_a = \psi_a^\dagger \psi_a$$

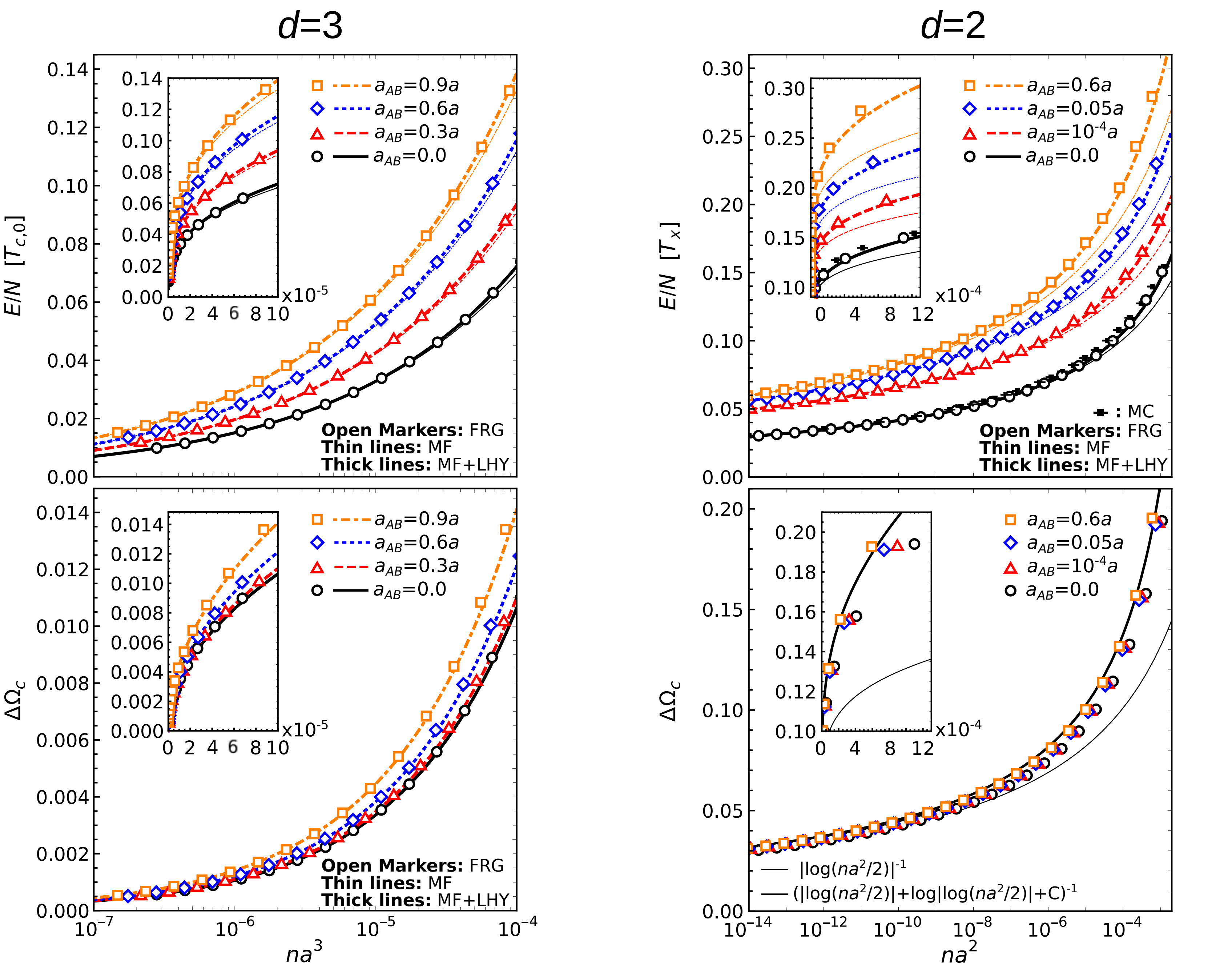
intra-species

inter-species

Results: Bose-Bose mixture



Results: Bose-Bose mixture

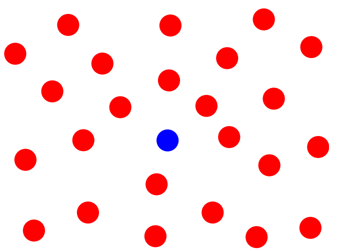


MC: G. Astrakharchik *et al.*, PRA **79**, 051602(R) (2009)

FRG for repulsive Bose-Bose mixtures

- FRG calculations compare favourably with known macroscopic results
- Future work:
  - Finite temperature
  - Spin drag
  - **Attractive Bose-Bose mixture**: liquid phase, dimerization, strongly-interacting regime

The Bose polaron



- Impurity immersed in a weakly-interacting Bose gas
- Several developments in the past few years:
  - Experimental realization

    N. Jørgensen *et al.*, PRL **117**, 055302 (2016), M. Hu *et al.*, PRL **117**, 055301 (2016)
  - Theoretical description of strong coupling regime

    J. Levinsen et al., PRL **115**, 125302 (2015). N.-E. Guenther *et al.*, PRL **120**, 050405 (2018). L. Peña Ardila et al., PRA **99**, 063607 (2019).
- Bosonic medium: three- and more-body correlations can be important
- Efimov physics

  N. T. Zinner, EPL **101**, 60009 (2013). M. Sun *et al.*, PRL **119**, 013401 (2017)

The Bose polaron

- Microscopic action

$$\mathcal{S} = \int_x \Bigg[\psi_B^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_B} - \mu_B\right)\psi_B + \psi_I^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_I} - \mu_I\right)\psi_I + \frac{g_{BB}}{2}(\psi_B^\dagger \psi_B)^2 + g_{BI}\psi_B^\dagger \psi_I^\dagger \psi_B \psi_I\Bigg]$$

μ_B: chemical potential of the medium
μ_I: polaron energy

- Boson-impurity interaction can either be repulsive or attractive
- Resonant interaction in attractive branch: $a_{BI} \to \infty$
- Hubbard-Stratonovich transformation: **Dimer fields** $\phi \sim \psi_B \psi_I$

$$\mathcal{S} = \int_x \Bigg[\psi_B^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_B} - \mu_B\right)\psi_B + \psi_I^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_I} - \mu_I\right)\psi_I + \nu_\phi \phi^\dagger \phi + \frac{g_{BB}}{2}(\psi_B^\dagger \psi_B)^2 + h_\Lambda\left(\phi^\dagger \psi_B \psi_I + \phi \psi_B^\dagger \psi_I^\dagger\right)\Bigg]$$

S. P. Rath and R. Schmidt, PRA **88**, 053632 (2013)

The Bose polaron with the FRG

- We propose the following ansatz

$$\Gamma_k = \int_x \Bigg[\psi_B^\dagger \left(S_B\partial_\tau - \frac{Z_B}{2m_B}\nabla^2 - V_B\partial_\tau^2\right)\psi_B + \psi_I^\dagger\left(S_I\partial_\tau - \frac{Z_I}{2m_I}\nabla^2 + u_I\right)\psi_I$$

$$+U(\rho_B) + \phi^\dagger\left(S_\phi\partial_\tau - \frac{Z_\phi}{2m_\phi}\nabla^2 + u_\phi\right)\phi + h\left(\phi^\dagger\psi_B\psi_I + \phi\psi_B^\dagger\psi_I^\dagger\right)$$

$$+\lambda_{B\phi}\psi_B^\dagger\phi^\dagger\psi_B\phi + \ldots\Bigg]$$

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$$+\lambda_{B\phi}\psi_B^\dagger\phi^\dagger\psi_B\phi + \ldots\Big]$$

Boson-boson interaction

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Boson-boson interaction

Boson-impurity interaction

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Boson-boson interaction

Boson-impurity interaction

Three-body coupling

The Bose polaron with the FRG

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$$+\lambda_{B\phi}\psi_B^\dagger\phi^\dagger\psi_B\phi + \ldots\Big]$$

Boson-boson interaction

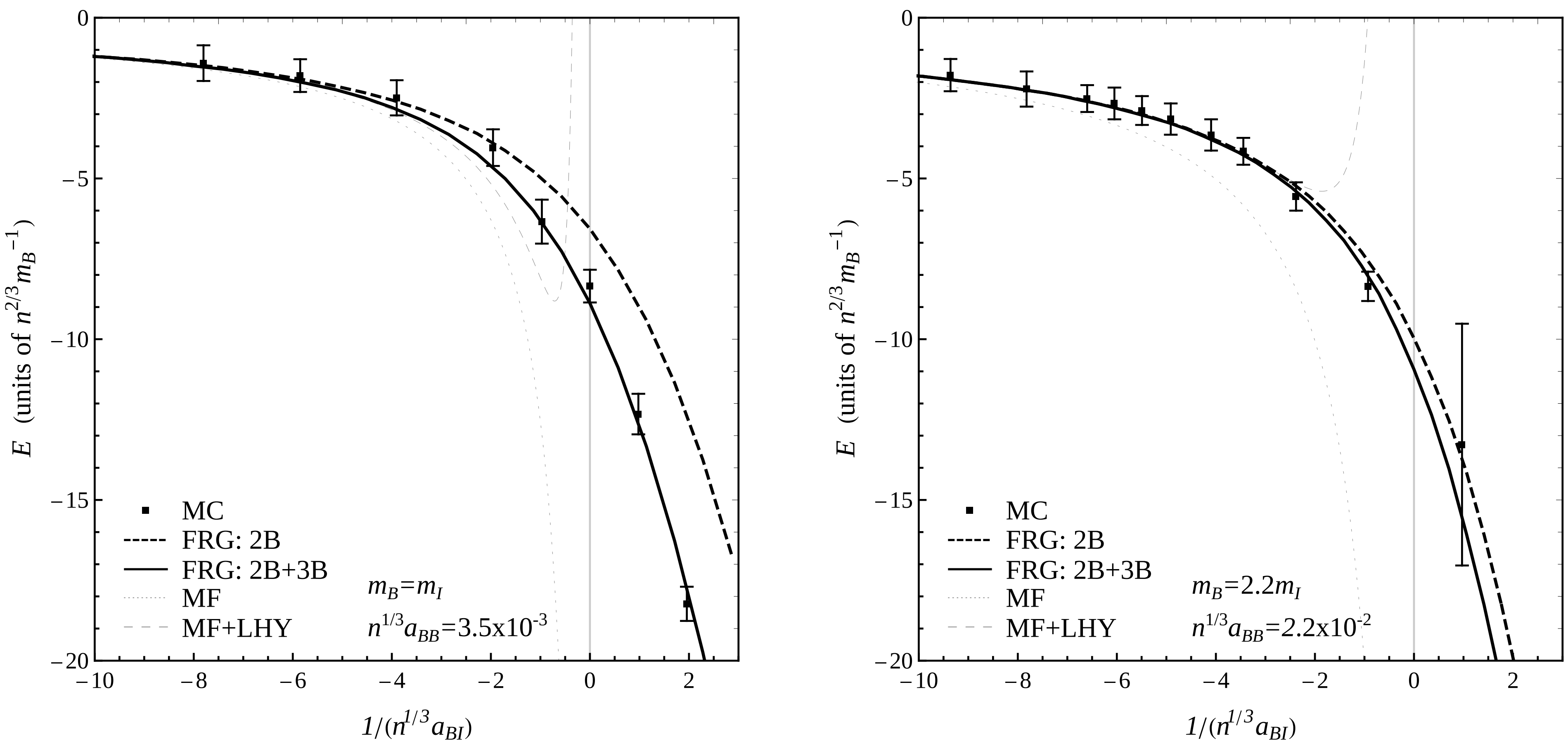
Boson-impurity interaction

Three-body coupling

- Conceptually easy to add three- and more-body correlations

Results: Bose polaron (d=3)

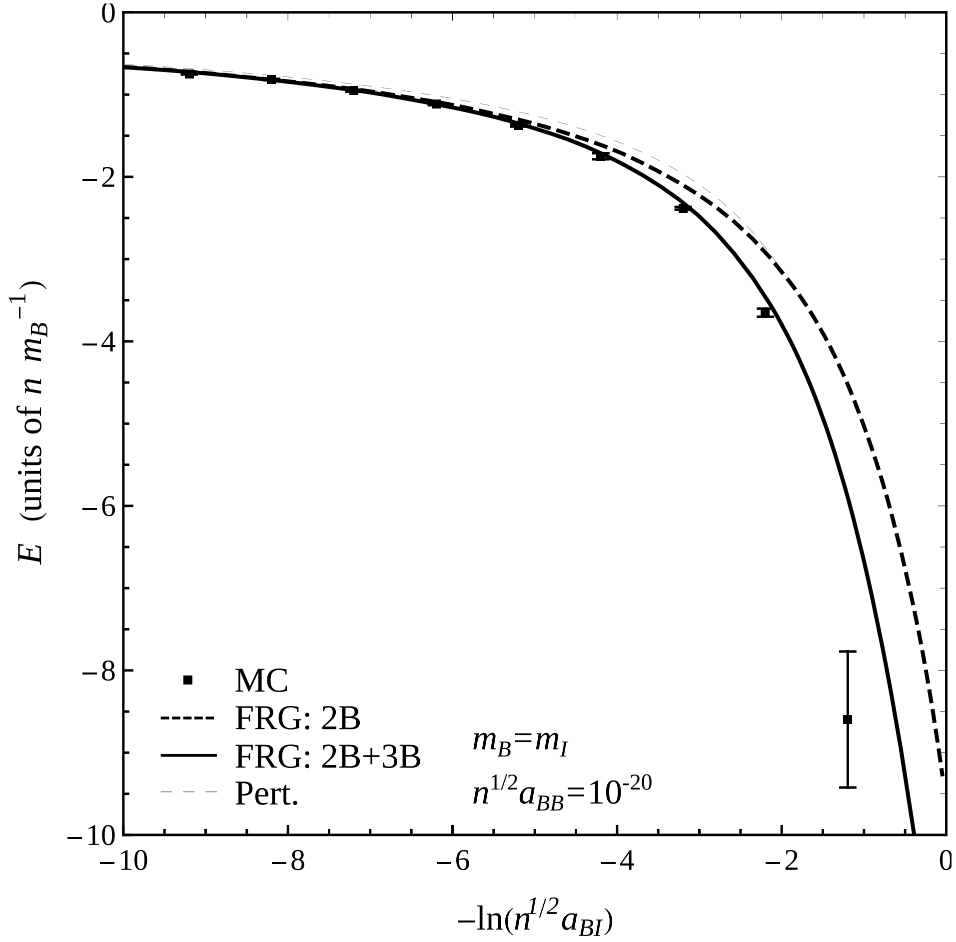
<u>**Work in progress**</u>



<u>MC</u>: L. Peña Ardila *et al.*, PRA **A** 99, 063607 (2019)

Results: Bose polaron (d=2)

Work in progress



MC: L. Peña Ardila *et al.*, PRR **2**, 023405 (2020)

Conclusions

- The FRG can provide a successful description of bosonic mixtures.
- It gives a good description of the ground state properties of the **Bose polaron** within a derivative expansion. Strong coupling regime is reasonably well described.
  - Future work: **Finite temperature**, four-body correlations, momentum dependence
- Macroscopic properties of repulsive Bose-Bose mixtures are well described.
  - Future work: **Attractive Bose-Bose mixtures**
- Other related extensions: Bose-Fermi mixtures, SU(N) Fermi gases

Example: Weakly interacting Bose gas

- In a field theory formulation, a weakly interacting Bose gas is described by the microscopic action

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Ω: grand-canonical potential

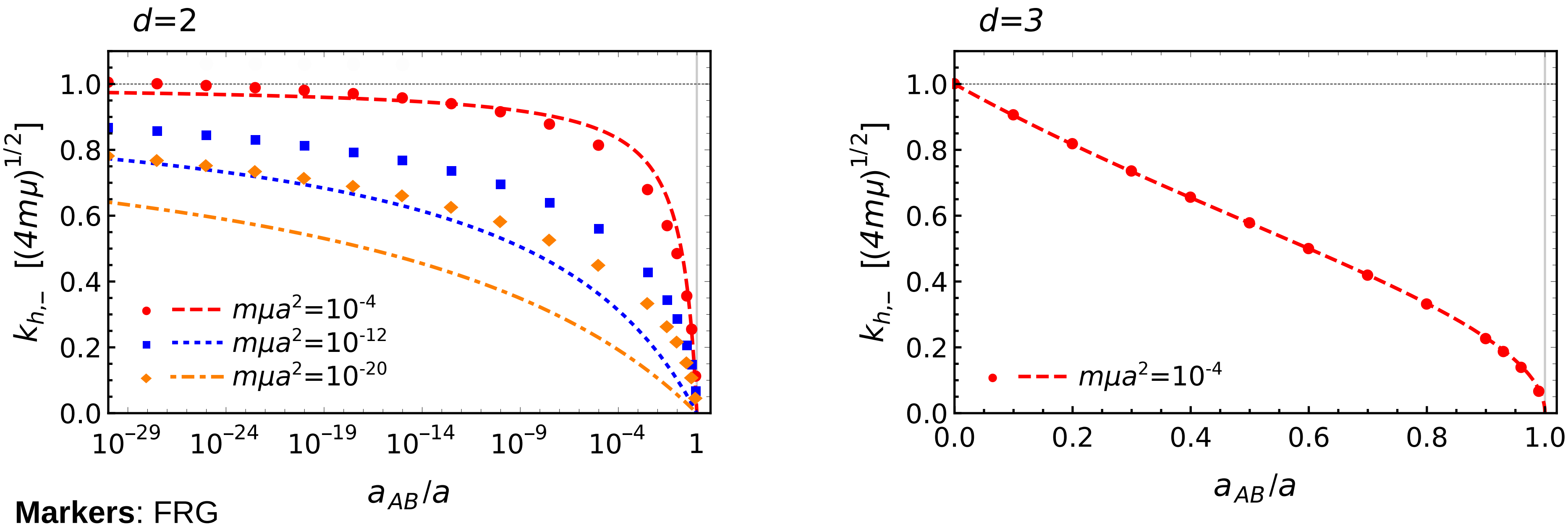
- The interaction is related to the two-body *T* matrix

$$T^{2B} = \begin{cases} \frac{4\pi a}{m} & : d = 3 \\ \frac{4\pi/m}{\log(2/|\mu| a^2) - 2\gamma_E} & : d = 2 \end{cases}$$

a: s-wave scattering length

Results: Bose-Bose mixture

- A vanishing spin healing scale $k_{h,-}$ signals the phase separation



Markers: FRG
Lines: Bogoliubov

- Within the range of parameters explored, we find that the point of phase separation is $a_{AB} = a$ (mean field result).

- RG studies suggest that the phase separation point in two dimensions occurs for $a_{AB} < a$ at logarithmically small densities

A. K. Kolezhuk, PRA **81**, 013601(2010)

The Bose polaron with the FRG

- We propose the following ansatz

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$$+\lambda_{B\phi}\psi_B^\dagger \phi^\dagger \psi_B \phi + \ldots\Bigg]$$

- No feedback from the impurity

- Drawbacks of the derivative expansion:
  - Not really suitable to study few-body physics: big mass imbalance? Efimov trimers?
  - Spectral function is not accessible