Dipolar atomtronics circuits: magnetostirring and quantum control

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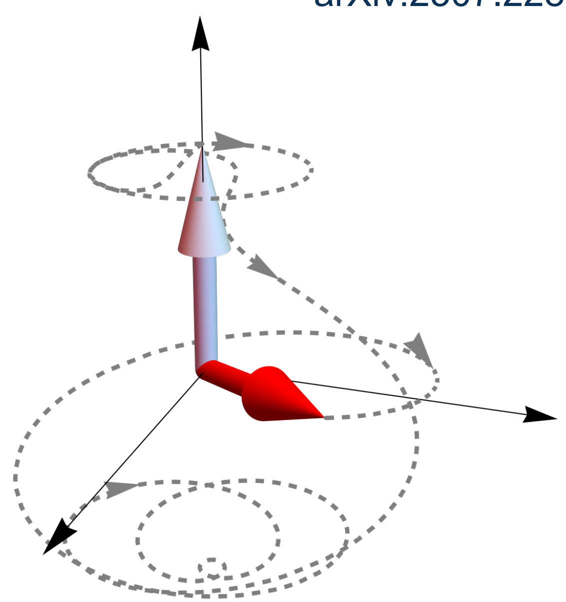
arXiv:2507.22822 (2025).



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Excelencia María de Maeztu 2020–2024

Dipolar atomtronics circuits

1. **Ultracold atoms** and **atomtronics**.
2. Ultracold **dipolar gases**.
3. Dipolar **magnetostirring** protocol for **atomtronics circuits**.
4. Dipolar **optimal control** of quantum states.
5. Summary.

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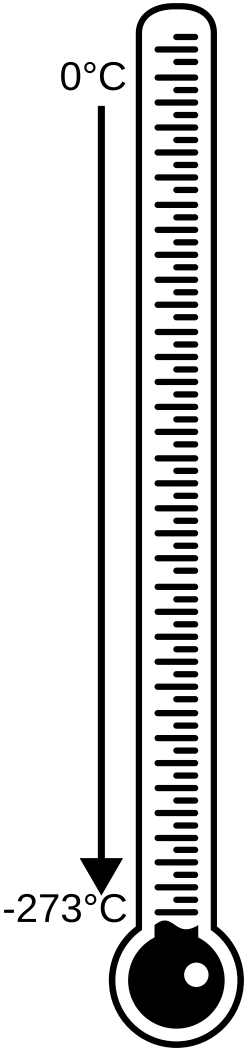
Ultracold atoms

- They are atoms that are **cooled** and **trapped** at **ultracold temperatures** (≤μK).
- At such low temperatures, **quantum effects become important**.
- Their realisation became possible thanks to progress in **cooling and trapping techniques** during the 80s.
  H. J. Metcalf and P. van der Straten, *Laser cooling and trapping,* Springer Science & Business Media (1999).
  - ➔ **Laser cooling**, magneto-optical traps (MOT), etc.
  - ➔ **1997**: **Nobel Prize** was awarded to S. Chu, C. Cohen-Tannoudji, and W. D. Phillips.
    W. D. Phillips, Rev. Mod. Phys. **70**, 721 (1998).

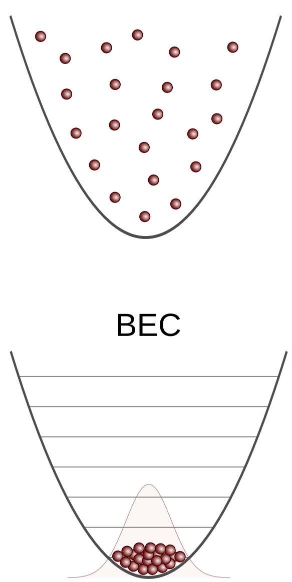


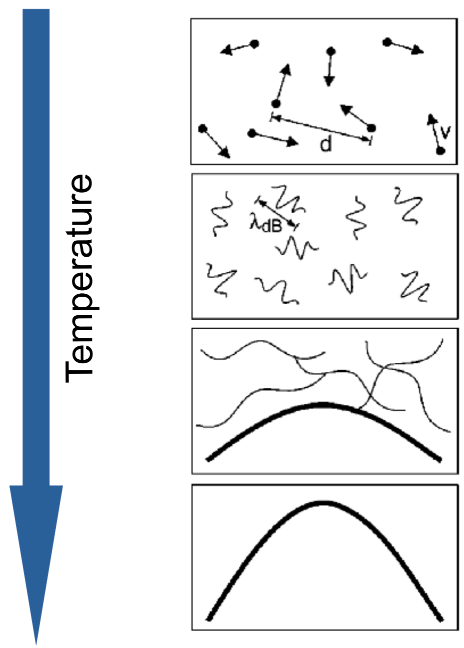
Bose-Einstein condensates (BECs)

- **BECs** were **realised experimentally** for the first time in **1995** with a gas of **ultracold bosonic atoms**.
  JILA: M. H. Anderson *et al.*, Science **269**, 198 (1995). **MIT**: K. B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995).

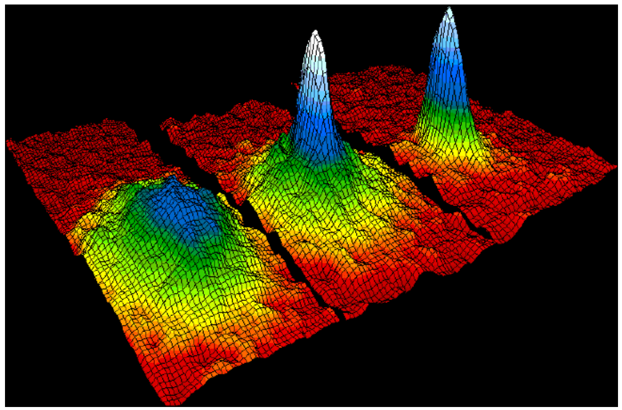
  → **2001:** **Nobel Prize** was awarded to E. Cornell, C. Wieman, and W. Ketterle.
  E. Cornell and C. E. Wieman, Rev. Mod. Phys. **74**, 875 (2002). W. Ketterle, Rev. Mod. Phys. **74**, 1131 (2002).

- A **BEC** corresponds to a **state of matter** where **bosons macroscopically** condense into their **ground state**.





Extracted from Rev. Mod. Phys. **74**, 1131 (2002).



Velocity distribution of a gas of ^{87}Rb atoms.

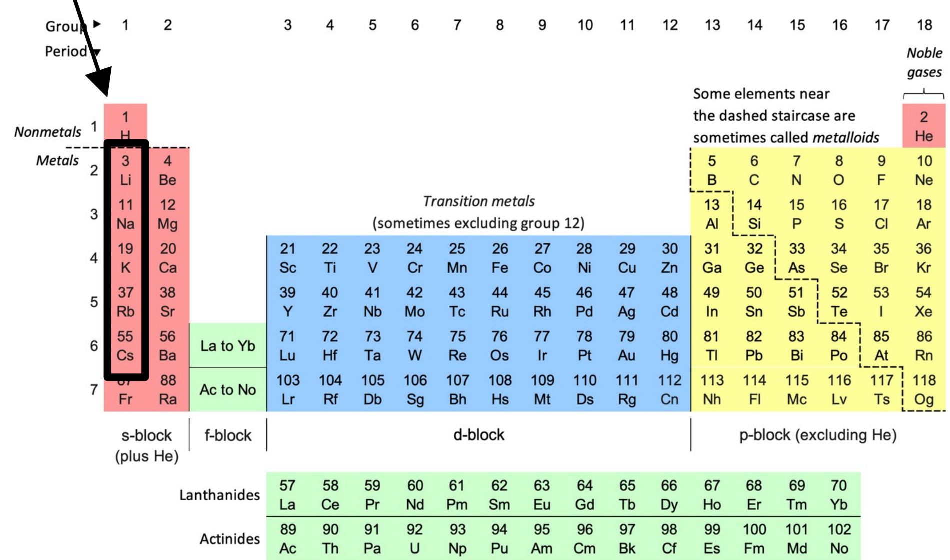
→ **Superfluid**

Isotopes

- Most experiments use **alkaline atoms**.
- The **isotope** dictates the **statistics**:
  - Bosons: ^{7}Li , $\boxed{^{23}\text{Na}}$, $\boxed{^{87}\text{Rb}}$, ^{133}Cs

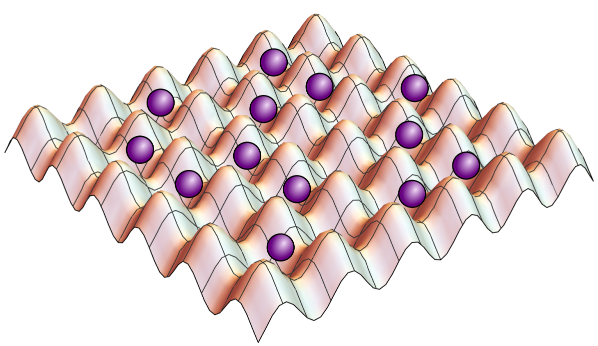
    BEC 95' MIT   BEC 95' JILA
  - Fermions: $\boxed{^{6}\text{Li}}$, $\boxed{^{40}\text{K}}$

    BCS 04' Innsbruck   BCS 04' JILA



Controllability

- Ultracold atoms offer an **unprecedented level** of **controllability**.
- The short-range **interatomic interaction** can be **tuned at will** via **Feshbach resonances**.
  C. Chin *et al.*, Rev. Mod. Phys. **82**, 1225 (2010).
- Ultracold atoms can be **confined** into **different geometries**, and also into **optical lattices**.
  I. Bloch, Nat. Phys. **1**, 23 (2005)



Quantum simulation →

Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond

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(*Received 31 May 2006; in final form 11 January 2007*)

Atomtronics

- **Atomtronics** is an **emerging field** that aims to build coherent **matter-wave circuits** by manipulating **ultracold atoms**.
- It was initially proposed for building ultracold atomic **analogs** to traditional **electronic devices**.

PHYSICAL REVIEW A **75**, 023615 (2007)

Atomtronics: Ultracold-atom analogs of electronic devices

B. T. Seaman, M. Krämer, D. Z. Anderson, and M. J. Holland
JILA, National Institute of Standards and Technology and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440, USA
(Received 23 June 2006; published 20 February 2007)

- However, over the years, **atomtronics** has attracted increased interest in developing new **quantum technologies**.

New J. Phys. **19** (2017) 020201 https://doi.org/10.1

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IOP Institute of Physics

EDITORIAL

Focus on atomtronics-enabled quantum technologies

Luigi Amico[1,2], Gerhard Birkl[3], Malcolm Boshier[4] and Leong-Chuan Kwek[2,5]

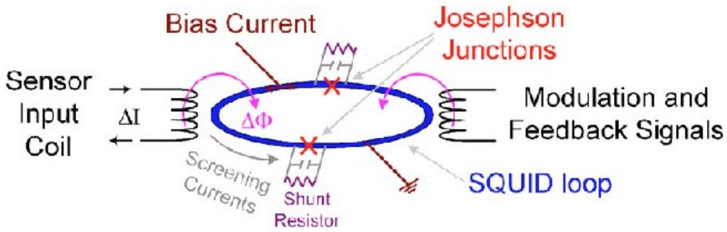
REVIEWS OF MODERN PHYSICS, VOLUME 94, OCTOBER–DECEMBER 2022

***Colloquium*: Atomtronic circuits: From many-body physics to quantum technologies**

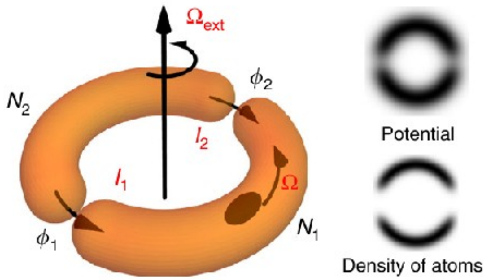
Luigi Amico*
Quantum Research Centre, Technology Innovation Institute, Abu Dhabi 9639, United Arab Emirates, INFN-Sezione di Catania, Via Santa Sofia 64, 95127 Catania, Italy, Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543, Singapore, and MajuLab, CNRS-UNS-NUS-NTU International Joint Research Unit, UMI 3654, Singapore 117543, Singapore

Atomtronics

- A notorious example within atomtronics is the **atomic analogs** of **Superconducting Quantum Interference Devices** (SQUIDs).
  C. Ryu, P. W. Blackburn, A. A. Blinova, and M. G. Boshier, Phys. Rev. Lett. **111**, 205301 (2013).



dc SQUID
Extracted from Rev. Sci. Instrum. **77**, 101101 (2006)



AQUID
Extracted from Nat. Commun. **11**, 3338 (2020)

→ Flux qubits.
D. Aghamalyan *et al.*, New J. Phys. **18**, 075013 (2016).

→ **Quantum sensors**, such as gyroscopes.
C. Ryu, E. C. Samson, M. G. Boshier, Nat. Commun. **11**, 3338 (2020).

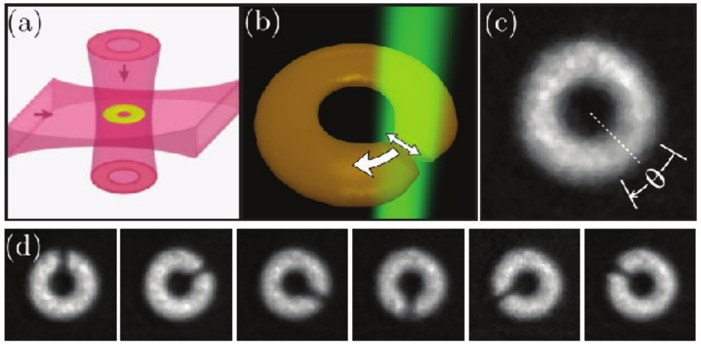
- For comprehensive and “recent” reviews on atomtronics, please check L. Amico *et al.*, AVS Quantum Sci. **3**, 039201 (2021) and L. Amico *et al.*, Rev. Mod. Phys. **94**, 041001 (2022).

Atomtronics rings and persistent currents

- **Ring circuits** can be realised by trapping ultracold atoms in **toroidal traps**.
  C. Ryu, M. F. Andersen, P. Cladé, V. Natarajan, K. Helmerson, W. D. Phillips, Phys. Rev. Lett. **99**, 260401 (2007).

- A fundamental aspect of atomtronics circuits is the **generation** of **persistent circulation**.
  J. Polo, W.J. Chetcuti, T. Haug,.A. Minguzzi, K. Wright, L. Amico, Phys. Rep. **1137**, 1 (2025).

- There are different ways to **induce persistent circulation** (e.g., **barrier stirring**).
  K. C. Wright et al., Phys. Rev. Lett. **110**, 025302 (2013).



Barrier stirring
Extracted from Phys. Rev. Lett. **110**, 025302 (2013).

We propose a **new method** for **generating circulation** using **dipolar gases**.

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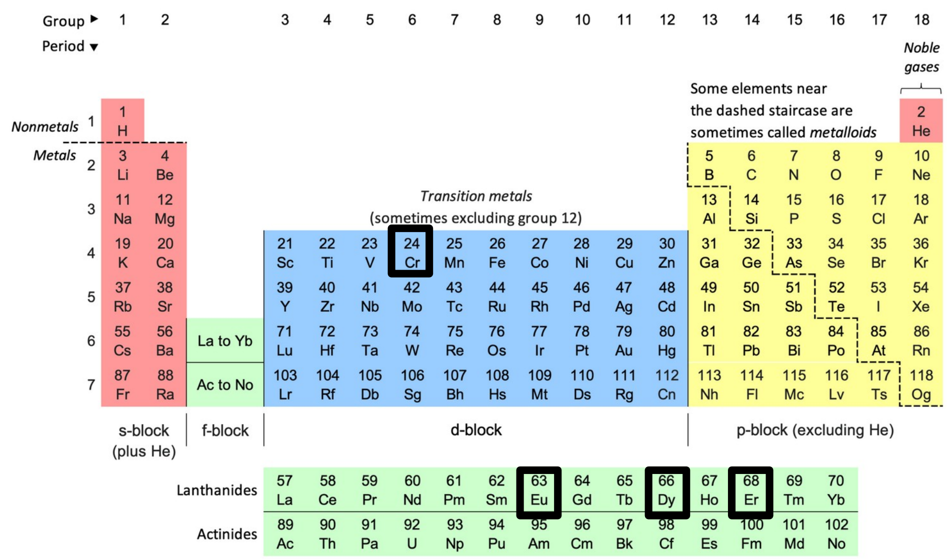
Magnetic atoms

- It is also possible to cool **magnetic atoms**, which have a **permanent magnetic dipole moment**.

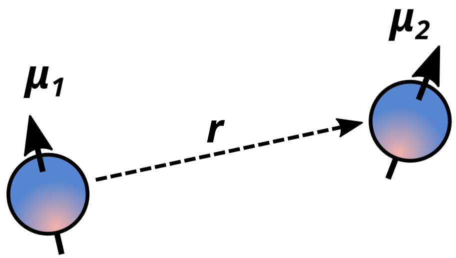


- **BECs** of such atoms have been realised over the years.

Cr: A. Griesmaier, J. Werner, S. Hensler, J. Stuhler, T. Pfau, Phys. Rev. Lett. **94**, 160401 (2005).
Dy: M. Lu, N. Q. Burdick, S. H. Youn, B. L. Lev, Phys. Rev. Lett. **107**, 190401 (2011).
Er: K. Aikawa, A. Frisch, M. Mark, S. Baier, A. Rietzler, R. Grimm, F. Ferlaino, Phys. Rev. Lett. **108**, 210401 (2012).
Eu: Y. Miyazawa, R. Inoue, H. Matsui, G. Nomura, M. Kozuma, Phys. Rev. Lett. **129**, 223401 (2022).

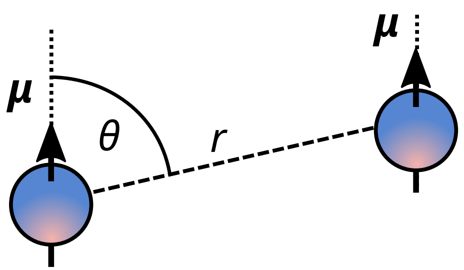
Dipole-dipole interaction

- The interaction between dipoles is **long-range** and **anisotropic**:



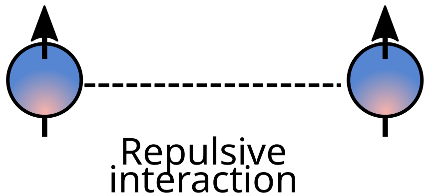
$$U_{\mathrm{dd}} \propto \frac{(\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2) r^2 - 3(\boldsymbol{\mu}_1 \cdot \boldsymbol{r})(\boldsymbol{\mu}_2 \cdot \boldsymbol{r})}{r^5} \, .$$

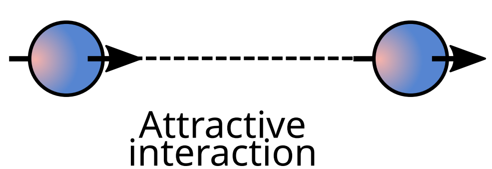
- If the dipoles are **polarised** in the **same direction**:



$$U_{\mathrm{dd}} \propto \frac{1 - 3\cos\theta}{r^3} \mu^2 \, .$$

- **Side-by-side** dipoles are **repulsive**, while **head-to-tail** dipoles are **attractive**.



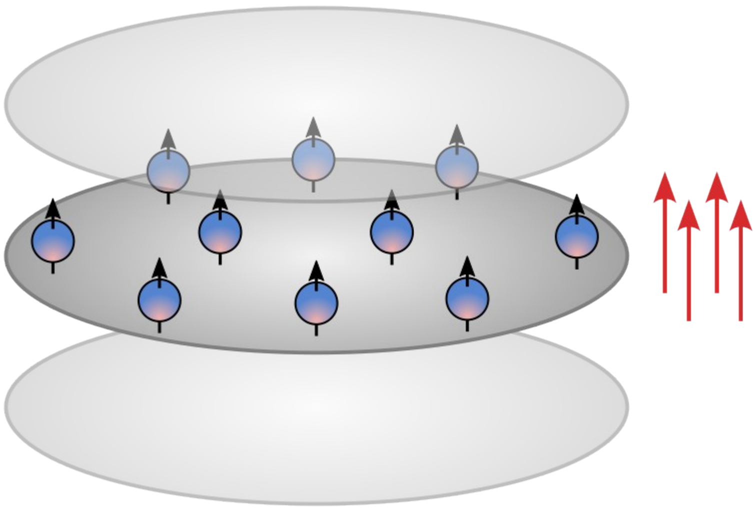


Dipolar gases

- Ultracold **dipolar gases** are influenced by the **dipole-dipole interactions** between particles.

M. A. Baranov, Phys. Rep. **464,** 71 (2008). T Lahaye *et al.*, Rep. Prog. Phys. **72**, 126401 (2009). L. Chomaz *et al.*, Rep. Prog. Phys. **86**, 026401 (2023).

- The **polarisation** can be **controlled** with **external magnetic fields**.



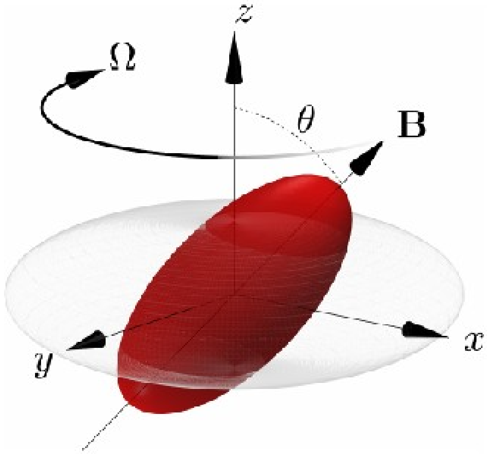
- The dipolar interaction produces **rich physics**, including crystalline phases, such as **supersolids.**

A. Recati, S. Stringari, Nat. Rev. Phys. **5**, 735 (2023).

Magnetostirring

- Magnetostirring is a technique where the **polarisation** of the dipoles is rotated.

S. B. Prasad *et al.* Phys. Rev. A **100**, 023625 (2019). T. Bland *et al.* Comptes Rendus. Physique **24**, S3, 133 (2023)



Extracted from Comptes Rendus. Physique **24**, S3, 133 (2023).

- It has been used to generate vortices in a dipolar condensate, which are a landmark **feature of superfluidity**.

L. Klaus, T. Bland, E. Poli, C. Politi, G. Lamporesi, E. Casotti, R. N. Bisset, M. J. Mark and F. Ferlaino, Nat. Phys. **18**, 1453 (2022).

- We propose the use of magnetostirring for **generating persistent circulation**.

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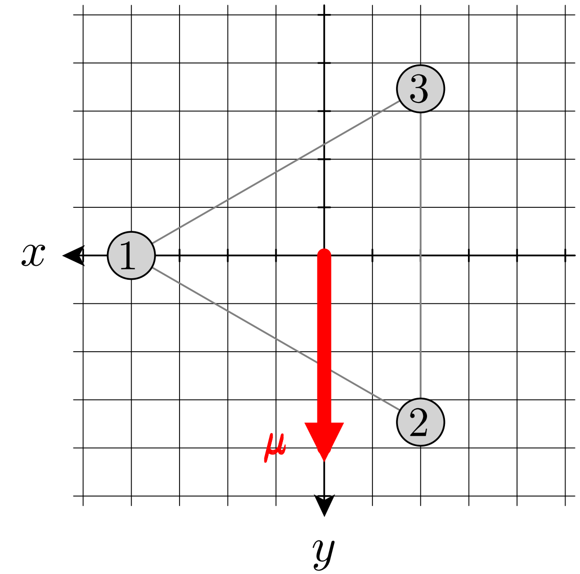
Dipolar bosons in a three-well circuit

- We consider **polar bosons** confined in a **ring** with **three sites**.
- We model this with an **extended Bose-Hubbard Hamiltonian**:

C. Trefzger, C. Menotti, B. Capogrosso-Sansone, M. Lewenstein, J. Phys. B **44**, 193001 (2011).



$$\hat{H} = -J\sum_{j=1}^{3}(\hat{a}_{j+1}^{\dagger}\hat{a}_j + \hat{a}_j^{\dagger}\hat{a}_{j+1}) + \frac{U}{2}\sum_{j=1}^{3}\hat{n}_j(\hat{n}_j - 1) + \sum_{j=1}^{3}\sum_{k\neq j}^{3}\frac{V_{jk}}{2}\hat{n}_j\hat{n}_k$$

Tunnelling | On-site repulsive | Long-range dipolar interaction

Repulsive on-site: $U > 0$

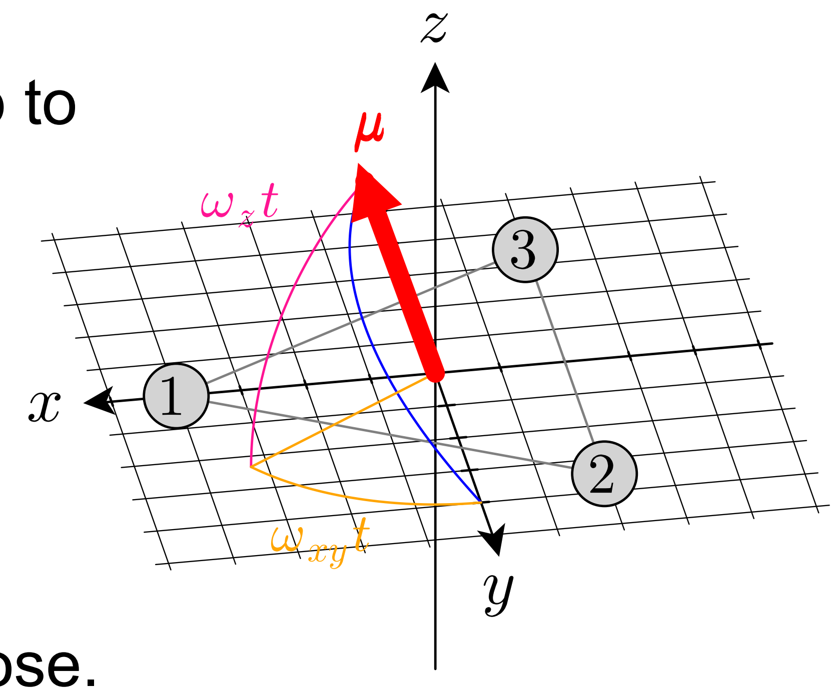
Dipolar interaction: $V_{jk} = \frac{U_d}{|\mathbf{r}_j - \mathbf{r}_k|^3}\left\{1 - 3\left[\frac{\boldsymbol{\mu}\cdot(\mathbf{r}_j - \mathbf{r}_k)}{|\mathbf{r}_j - \mathbf{r}_k|}\right]^2\right\}$

- Each calculation will consider a **fixed number** N of **bosons**.

Circulation creation protocol

- **Protocol**: The **polarisation** from $t=0$ up to $t_f=\pi/(2\omega_z)$ **evolves** as

$$\boldsymbol{\mu}\cdot\mathbf{e}_x = \sin(\omega_{xy}\,t)\cos(\omega_z\,t)\,,$$
$$\boldsymbol{\mu}\cdot\mathbf{e}_y = \cos(\omega_{xy}\,t)\cos(\omega_z\,t)\,,$$
$$\boldsymbol{\mu}\cdot\mathbf{e}_z = \sin(\omega_z\,t)\,,$$



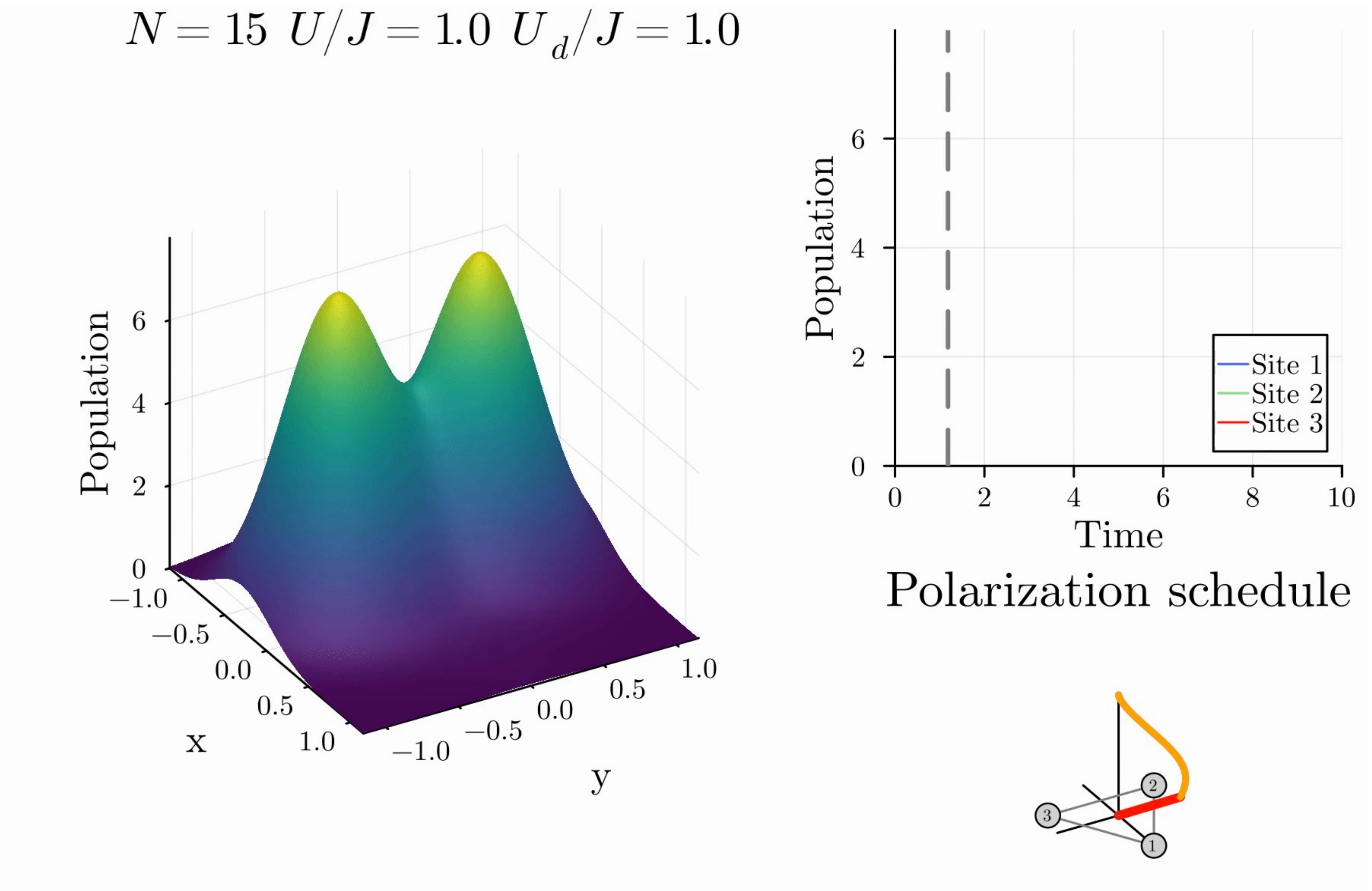
where ω_{xy} and ω_z are parameters to choose.

- **After the protocol** ($t>t_f$), the **polarisation** remains **fixed** in the z**-direction**.

- We choose $U=U_d$, so that **after the protocol** ($t>t_f$), the system becomes **non-interacting**.

- The **initial state** is chosen as the **ground state** for $t=0$.

Example of circulation

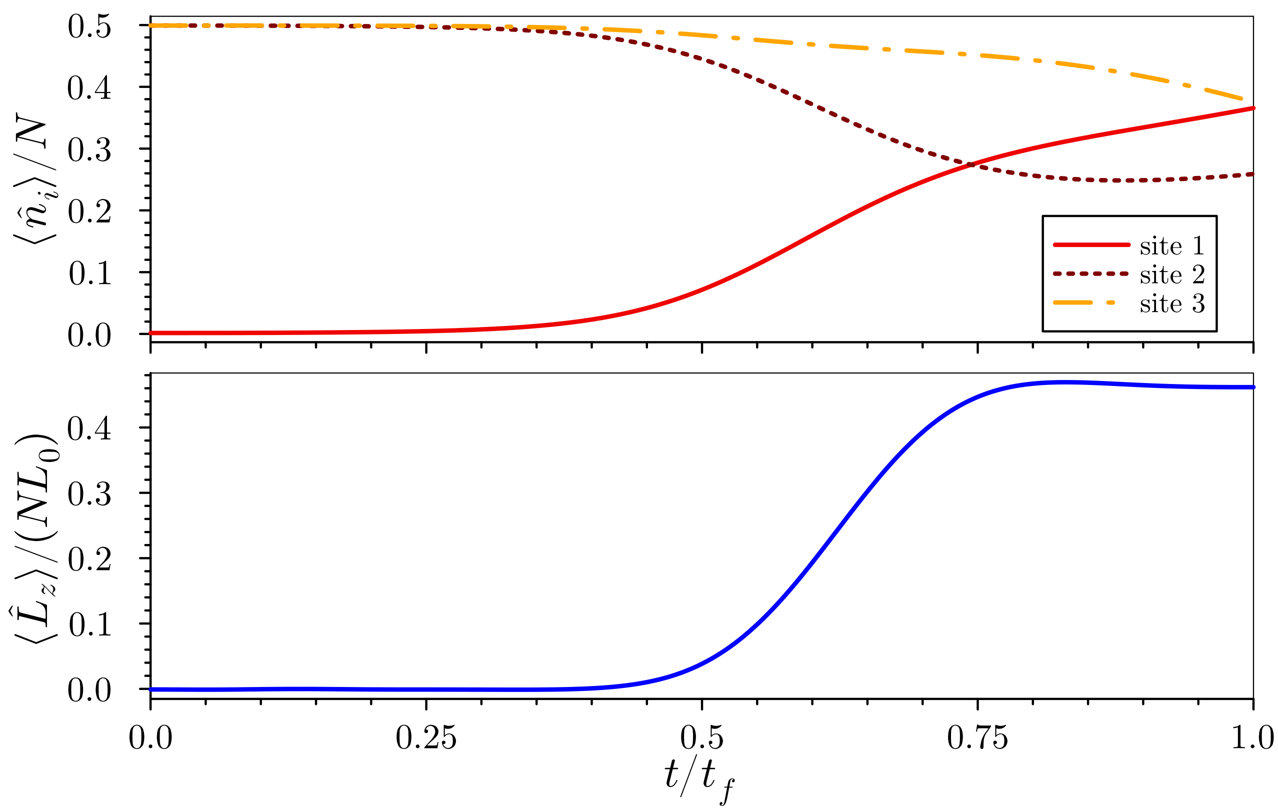


Circulation operator

- To measure the generation of circulation, we compute the **circulation operator**,

$$\hat{L}_z = i\,\frac{2\pi}{3}\,\frac{JmR^2}{\hbar}\,\sum_{j=1}^{3}\left(\hat{a}^\dagger_{j+1}\hat{a}_j - \hat{a}^\dagger_j\hat{a}_{j+1}\right).$$

R : circuit's radius



$N = 8$

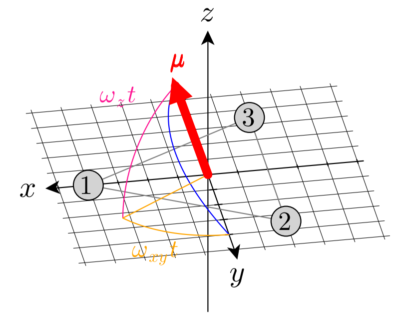
$U = U_d = 5$

$\omega_{xy} = 1.07\hbar/J$

$\omega_z = 1.32\hbar/J$

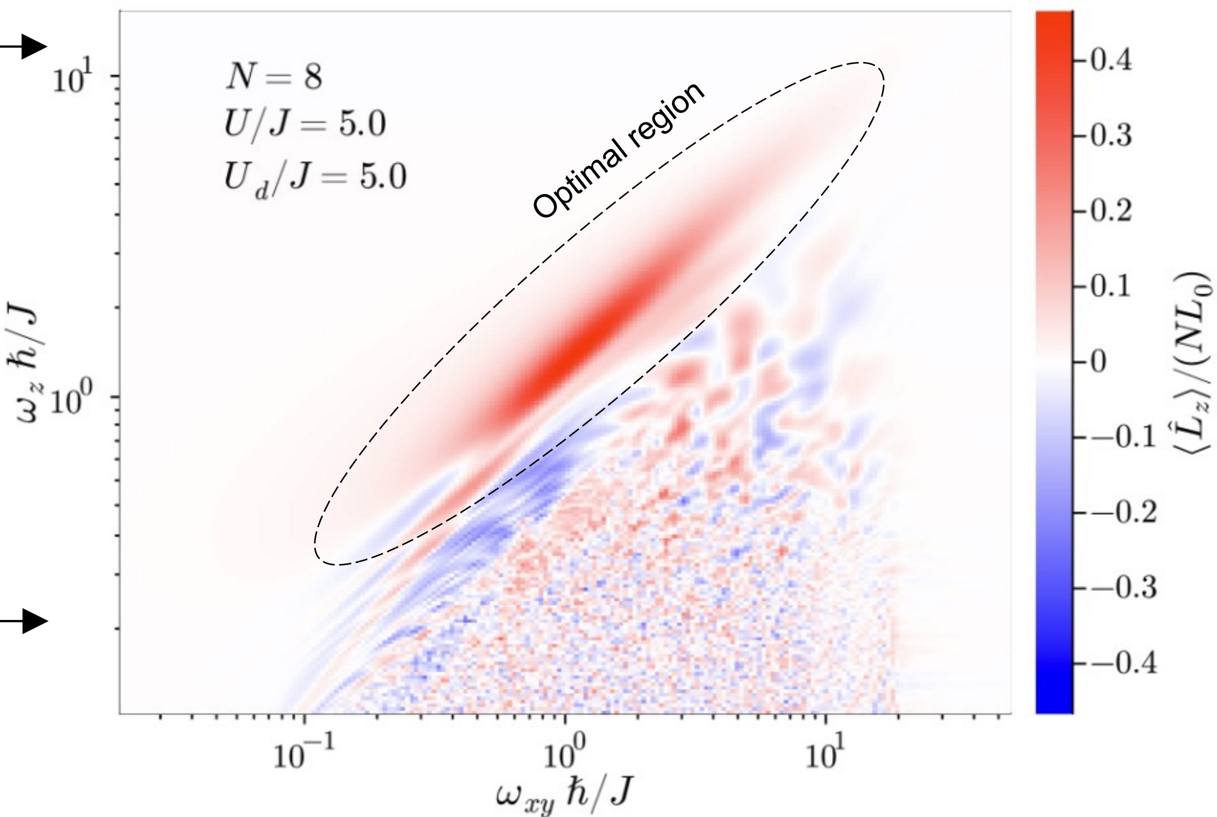
L_0 : max. possible circulation

Parameter optimisation



Too short

Too long



Too slow

Too fast

$t_f = \pi/(2\omega_z)$

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Quantum optimal control

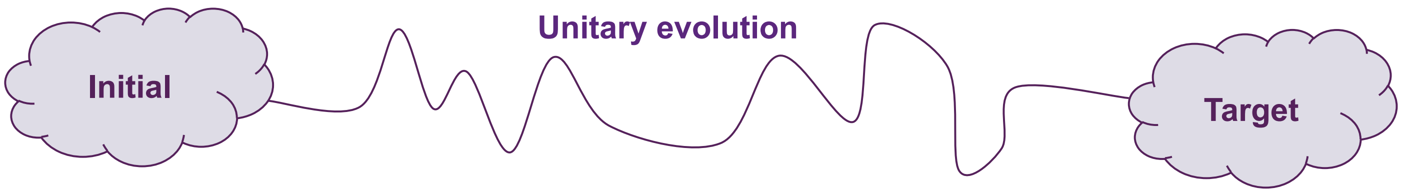
- The **spiral trajectory** may **not** be the **most optimal** one for generating **circulation**.

- We use **quantum optimal control** (QOC) theory for finding **optimal trayectories**.
  D. Dong, I. Petersen, IET Control Theory & Applications **4**, 2651 (2010).
  D. D'Alessandro, *Introduction to Quantum Control and Dynamics* (Chapman and Hall/CRC, 2021).

- QOC enables us to **manipulate external fields** to **drive** a quantum system to a **target state**.

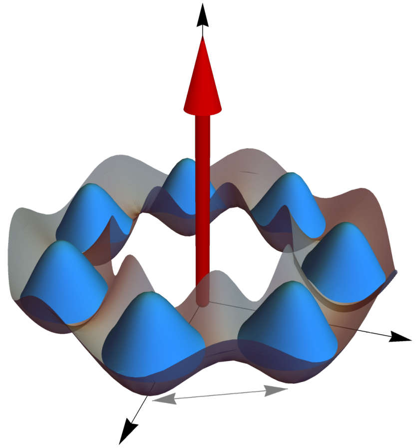
$$\hat{\mathcal{H}}(t) = \underbrace{\hat{H}_0}_{\text{Drift part}} + \underbrace{\hat{H}_c(\boldsymbol{\theta}, t)}_{\text{Control Hamiltonian}}$$

$\boldsymbol{\theta}$: parameters to modulate



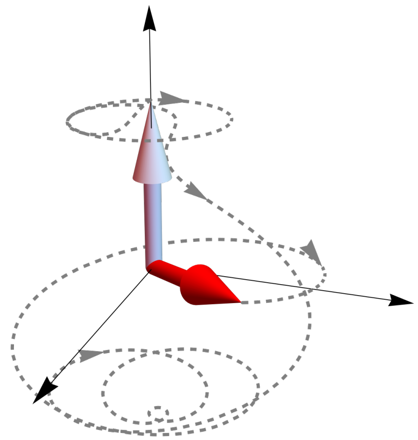
Dipolar bosons in a ring circuit

- This time, we consider **rings** with ***L* sites**.
- The **drift part**:



$$\hat{H}_0 = \sum_{j=1}^{L}\left[\underbrace{-J\left(\hat{a}^\dagger_{j+1}\hat{a}_j + \hat{a}^\dagger_j\hat{a}_{j+1}\right)}_{\text{Tunnelling}} + \underbrace{\frac{U}{2}\hat{n}_j\left(\hat{n}_j - 1\right)}_{\text{On-site interaction}}\right]$$

- The **control Hamiltonian**:



$$\hat{H}_c(\boldsymbol{\mu}, t) = \underbrace{U_d\sum_{\substack{j=1\\k>j}}^{L}\left[\frac{1}{|\mathbf{r}_{jk}|^3} - 3\frac{\left(\boldsymbol{\mu}(t)\cdot\mathbf{r}_{jk}\right)^2}{|\boldsymbol{\mu}|^2|\mathbf{r}_{jk}|^5}\right]\hat{n}_j\hat{n}_k}_{\text{Long-range dipolar interaction}}$$

Dipolar optimal control of entangled states

- We aim to generate selected **states** with **entangled circulation**.

T. Haug, R. Dumke, L.-C. Kwek, C. Miniatura, L. Amico, Phys. Rev. Res. **3**, 013034 (2021).

Target States: Entangled current states

$$\boxed{|\Psi_{\mathrm{EC}}\rangle = \frac{1}{\sqrt{|\Omega| N!}} \sum_{k\in\Omega} \left(\hat{b}_k^\dagger\right)^N |\mathrm{vac}\rangle}$$

Single-particle circular current

$$|k\rangle = \hat{b}_k^\dagger|\mathrm{vac}\rangle = \frac{1}{\sqrt{L}} \sum_{j=1}^{L} e^{i2\pi kj/L} \hat{a}_j^\dagger |\mathrm{vac}\rangle$$

where $\Omega=\{k_1,k_2,\ldots,k_{|\Omega|}\}$ is a set of $|\Omega|$ **winding numbers**.

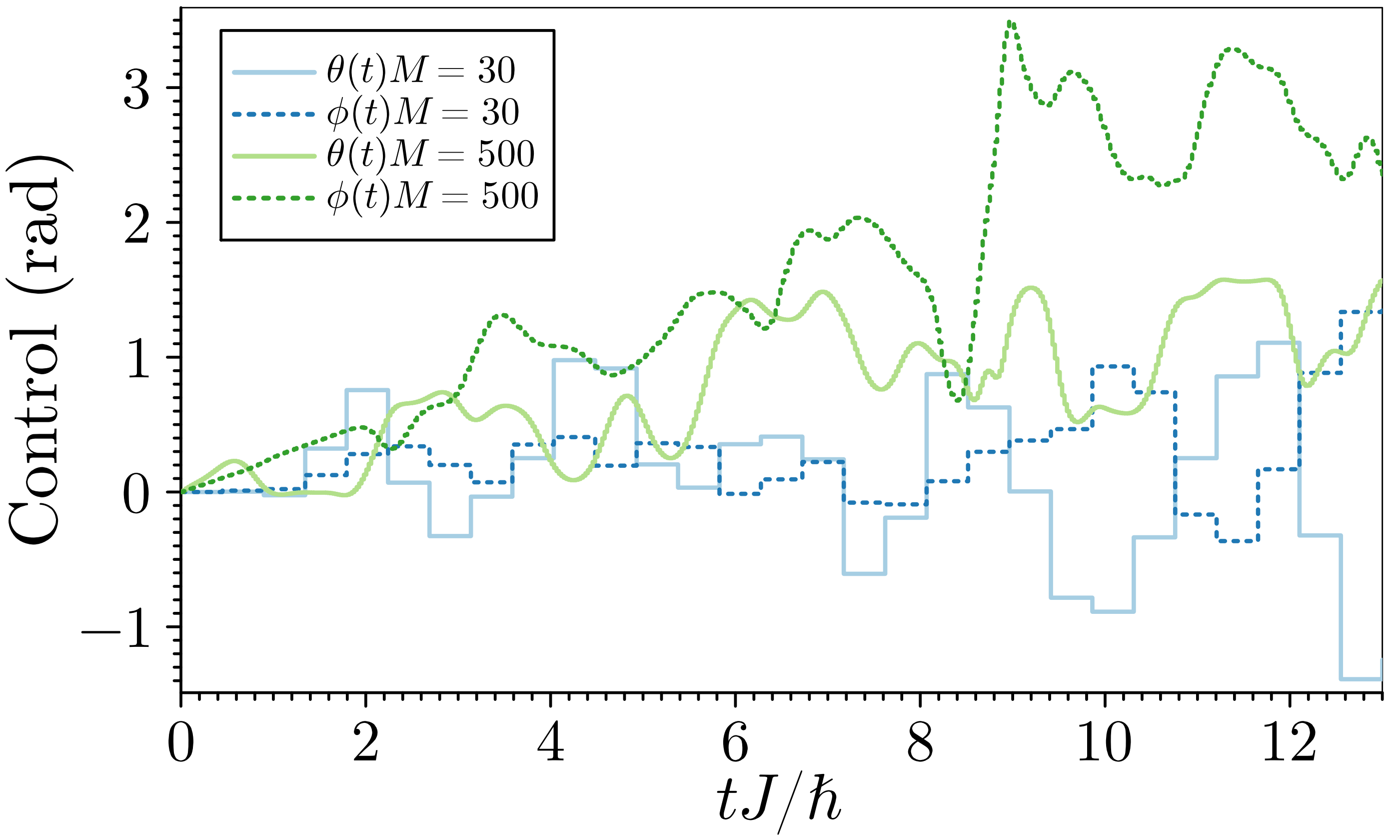
- We focus on the **NOON state** ($|\Omega|=2$) and the **W state** ($|\Omega|=3$).

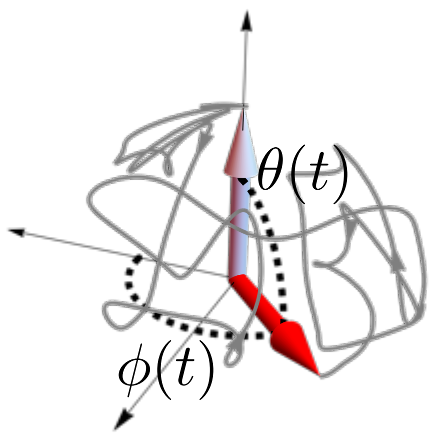
$$|\mathrm{NOON}\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle), \qquad |\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|N,0,0\rangle + |0,N,0\rangle + |0,0,N\rangle).$$

- We use the gradient-ascent pulse engineering (**GRAPE**) algorithm.

N. Khaneja *et al.*, Journal of Magnetic Resonance **172**, 296 (2005). M. H. Goerz *et al.*, Quantum **6**, 871 (2022),

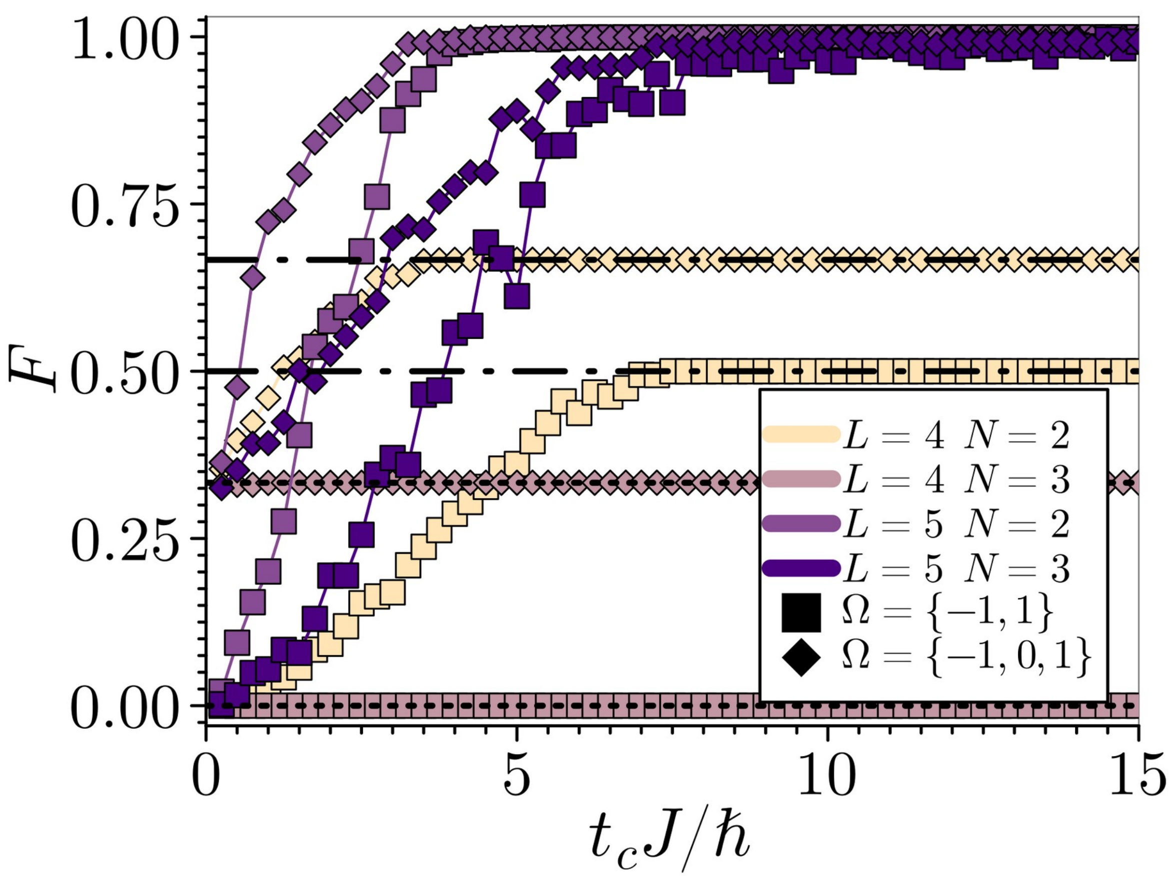
Optimised trajectories





M : control steps

Final fidelities



t_c : Protocols's time

- **Full fidelity** can be **achieved**, but for **only some configurations.**
- This is due to the **symmetries** of the system, which **constrain** the **achievable fidelity.**

    - For more details, please check arXiv: 2507.22822.

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2. Ultracold **dipolar gases**.
3. Dipolar **magnetostirring** protocol for **atomtronics circuits**.
4. Dipolar **optimal control** of quantum states.
5. Summary.

Summary

- **Magnetostirring** can be used to generate **persistent circulation** in **dipolar rings**.
- **Dipolar optimal control** (QOC) enables us to prepare **optimal protocols** for engineering **entangled currents**.
- These protocols can be **realised** by **controlling** the **polarisation** of the atoms with **external fields**.

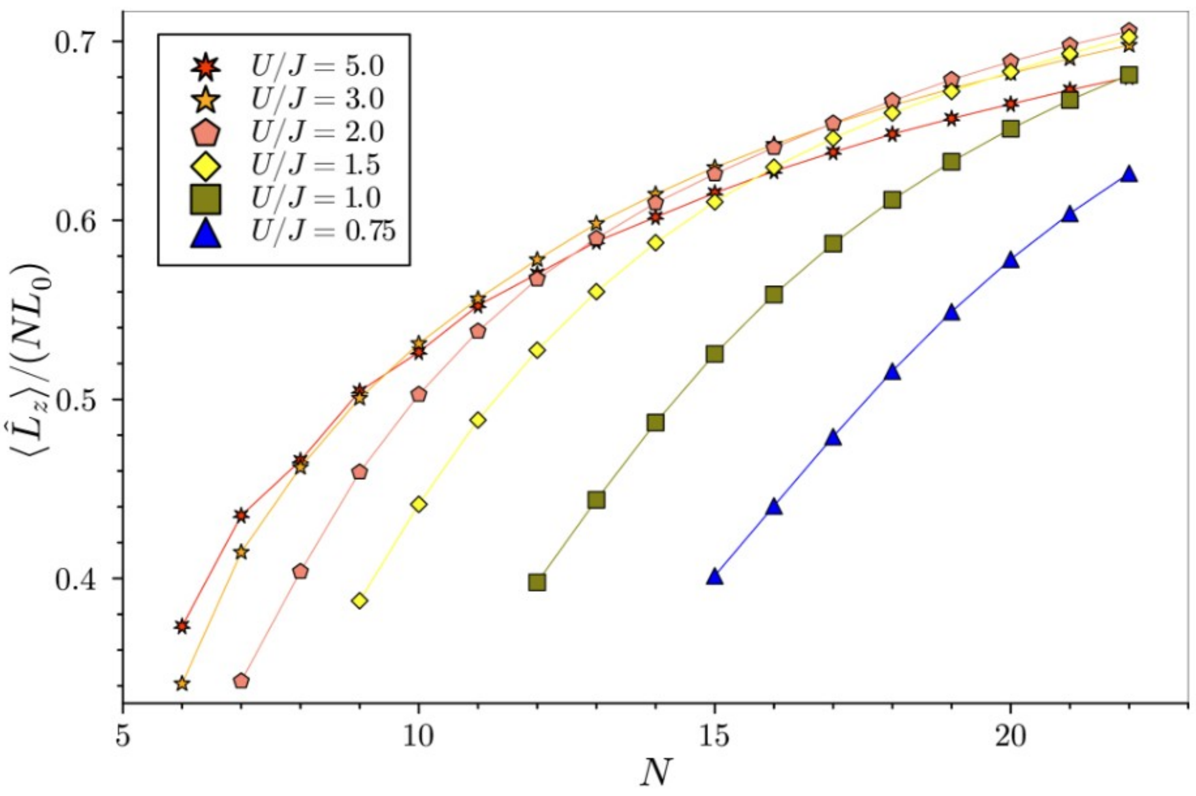
SciPost Phys. **19**, 059 (2025)

arXiv:2507.22822 (2025)

FONDECYT No. 3230023

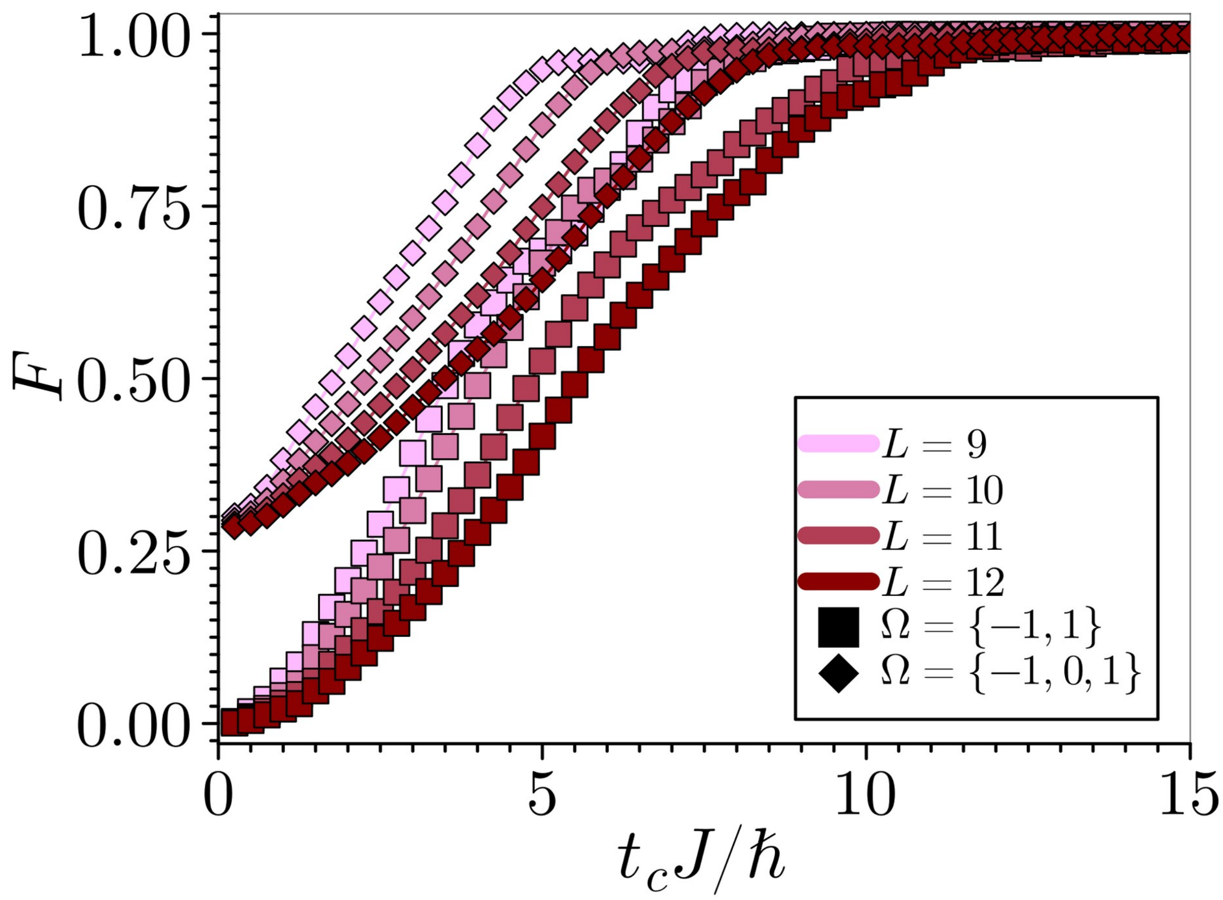
THANK YOU

Performance



- The **protocol** becomes **more efficient** with **more particles**.

Final fidelities



t_c : Protocols's time

- In **these cases** the system can achieve **full fidelity.**

Controllability summary

- **Symmetry constrains** provide **upper bounds** for the **fidelity**.

		Symmetric Rings (even *L*)		**Non-symmetric Rings** (odd *L*)
		Even *k*	Odd *k*	
Even number of bosons	*N* = 2	Limited by the DI eigenstate		Completely Controllable
	N > 2	Reachable	Reachable	
Odd number of bosons		Reachable	Unreachable	

Dipolar immune (DI) bound:

$$F_{\max} = 1 - |\langle \Psi_{\mathrm{DI}} | \Psi_{\mathrm{EC}} \rangle|^2$$
$$= 1 - \left| \frac{1}{\sqrt{|\Omega| L}} \sum_{k \in \Omega} (\delta_{k,L/4} + \delta_{k,3L/4}) \right|^2$$

Symmetry bound:

$$F_{\max} = \sum_{\substack{k \in \Omega \\ kN \text{ even}}} \frac{1}{|\Omega|}$$

- For more details please check arXiv: 2507.22822.