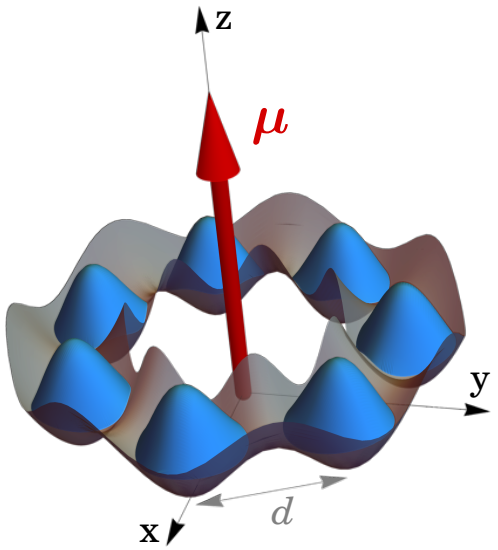
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FACULTAD DE FÍSICA
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Optimal control for dipolar Hubbard rings

Felipe Isaule
Departamento de Física
Universidad Técnica Federico Santa María

Physical Review Research **8**, 033121 (2026)



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24th November 2026

felipeisaule.github.io

felipe.isaule@usm.cl

Collaborators

Main author!



Héctor
Briongos Merino



Bruno
Juliá Díaz



Montserrat
Guilleumas

UNIVERSITAT DE BARCELONA

ICCUB | Institut de Ciències del Cosmos
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EXCELENCIA MARÍA DE MAEZTU 2020–2024

Quantum optimal control

- **Quantum optimal control** (QOC) provides a general methodology to engineer control protocols.

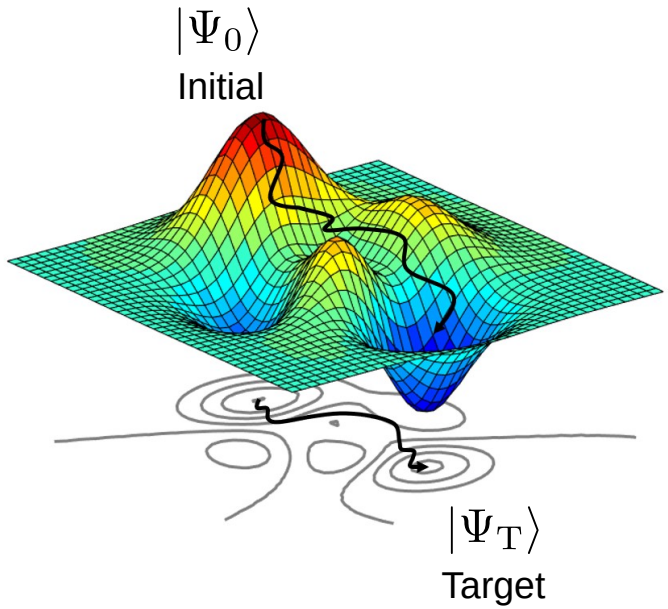
D. Dong, I. Petersen, IET Control Theory & Applications **4**, 2651 (2010).
D. D'Alessandro, *Introduction to Quantum Control and Dynamics* (Chapman and Hall/CRC, 2021).

- QOC enables us to **manipulate external fields** to **drive** a quantum system to a **target state**.

$$\hat{\mathcal{H}}(t) = \hat{H}_0 + \hat{H}_c(\boldsymbol{\theta}, t).$$

Drift part — Control Hamiltonian

θ : parameters to modulate



- We aim to maximise the **fidelity** over a **control time** t_c,

$$F[\theta(t)] = |\langle \Psi_T | U(\theta; t_c) | \Psi_0 \rangle|^2.$$

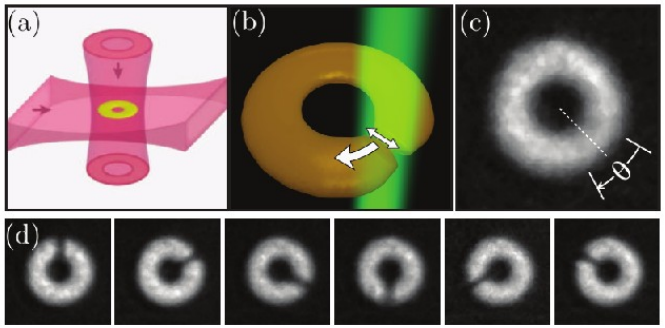
Atomtronics circuits

- **Atomtronics circuits** based on **ultracold atoms** offer a promising platform for testing QOC.
- Generating **persistent currents** in **ring geometries** is one of the main objectives in atomtronic devices.

  C. Ryu, M. F. Andersen, P. Cladé, V. Natarajan, K. Helmerson, W. D. Phillips, Phys. Rev. Lett. **99**, 260401 (2007).

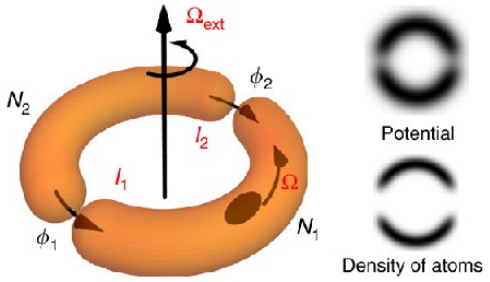
- They offer a high degree of **controllability** and potential for developing quantum devices.

  L. Amico *et al*., AVS Quantum Sci. **3**, 039201 (2021).
  L. Amico, D. Anderson, M. Boshier, J.-P. Brantut, L.-C. Kwek, A. Minguzzi, and W. von Klitzing, Rev. Mod. Phys. **94**, 041001 (2022).



Barrier stirring

Extracted from Phys. Rev. Lett. **110**, 025302 (2013).



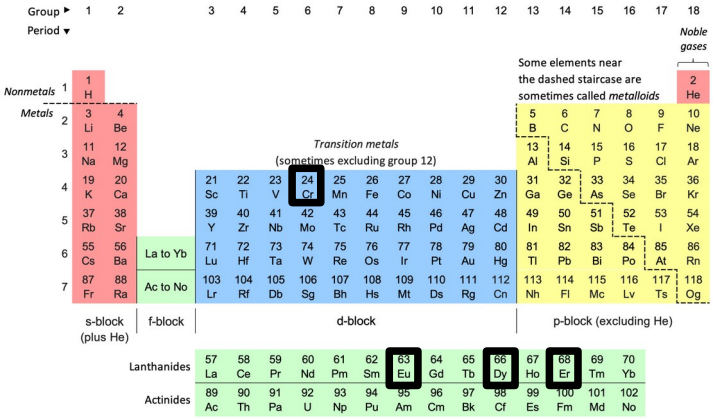
AQUID

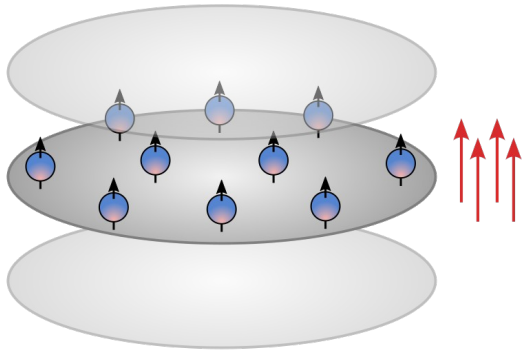
Extracted from Nat. Commun. **11**, 3338 (2020)

Dipolar gases

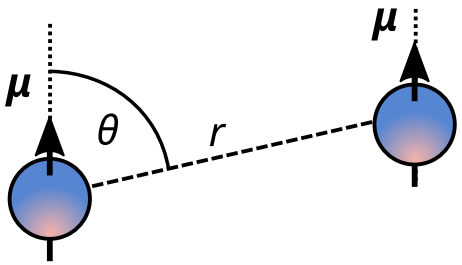
- It is possible to realise ultracold gases with **dipolar atoms**, which can be controlled through **external magnetic fields**.

T. Lahaye *et al*., Rep. Prog. Phys. **72**, 126401 (2009).
L. Chomaz *et al*., Rep. Prog. Phys. **86**, 026401 (2023).





- The interaction between dipoles is **long-range** and **anisotropic**. If the dipoles are **polarised** in the **same direction**,



$$U_{\mathrm{dd}} \propto \frac{1-3\cos\theta}{r^3}\mu^2 .$$

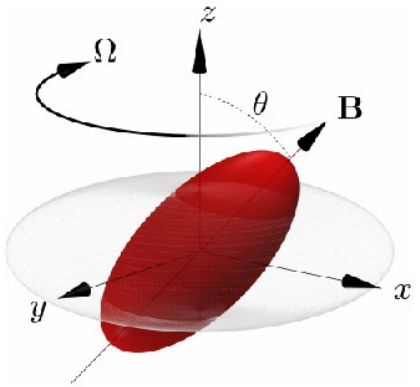
Magnetostirring

- **Magnetostirring** is a technique where the **polarisation** of the dipoles is **rotated**.

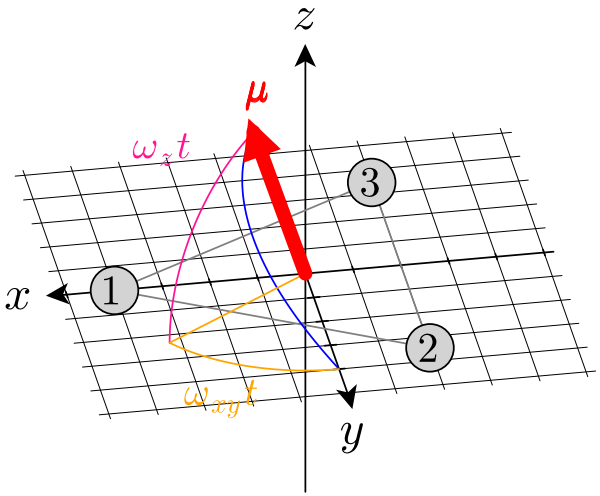
S. B. Prasad *et al.* Phys. Rev. A **100**, 023625 (2019).
T. Bland *et al.* Comptes Rendus. Physique **24**, S3, 133 (2023)

- It has been used to generate **vortices** in a dipolar condensate.

L. Klaus *et al.*, Nat. Phys. **18**, 1453 (2022).



Extracted from Comptes Rendus. Physique **24**, S3, 133 (2023).



- Magnetostirring can be used to **generate persistent currents** in **ring lattices**.

H. Briongos-Merino, F. Isaule, M. Guilleumas, and B. Juliá-Díaz, SciPost Phys. **19**, 059 (2025).

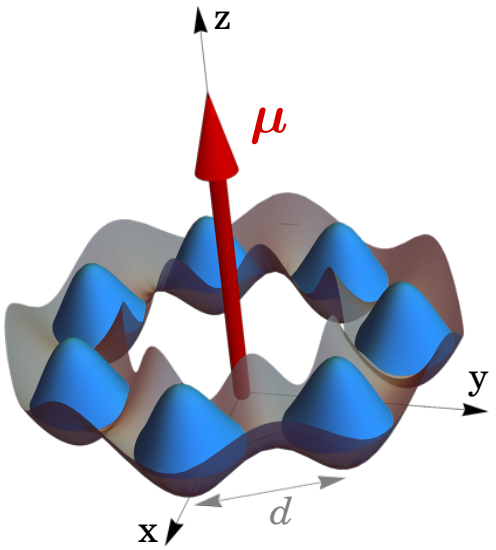
- We use **QOC** to generate **entangled currents** in dipolar ring lattices.

Model

- We consider **polar bosons** confined in a **ring lattice** with L **sites**,

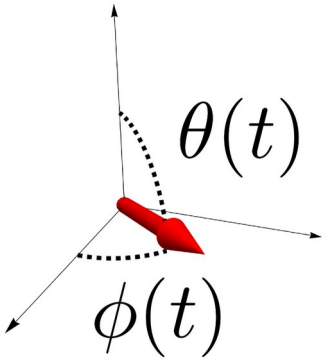
$$\hat{H}_0 = \sum_{j=1}^{L} \left[-J(\hat{a}_{j+1}^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_{j+1}) + \frac{U}{2}\hat{n}_j(\hat{n}_j - 1) \right],$$

Tunnelling — On-site interaction



$$\hat{H}_c(\boldsymbol{\mu}, t) = U_d \sum_{\substack{j=1 \\ k>j}}^{L} \left[\frac{1}{|\mathbf{r}_{jk}|^3} - 3\frac{(\boldsymbol{\mu}(t)\cdot\mathbf{r}_{jk})^2}{|\boldsymbol{\mu}|^2|\mathbf{r}_{jk}|^5} \right] \hat{n}_j\hat{n}_k.$$

Long-range dipolar interaction



- We **control** the **orientation of the dipoles**,

$$\boldsymbol{\theta}(t) = \{\theta(t), \phi(t)\}.$$

- The system is prepared in the **ground state** with $\theta{=}0$.

Dipolar optimal control of entangled currents

- We aim to generate **states** with **entangled circulation**,

T. Haug, R. Dumke, L.-C. Kwek, C. Miniatura, L. Amico, Phys. Rev. Res. **3**, 013034 (2021).

Target States: Entangled current states

$$|\Psi_{\mathrm{EC}}\rangle = \frac{1}{\sqrt{KN!}} \sum_{k\in\Omega} \left(\hat{b}_k^\dagger\right)^N |\mathrm{vac}\rangle,$$

Single-particle circular current

$$\hat{b}_k^\dagger|\mathrm{vac}\rangle = \frac{1}{\sqrt{L}} \sum_{j=1}^{L} e^{i2\pi kj/L} \hat{a}_j^\dagger|\mathrm{vac}\rangle,$$

where $\Omega=\{k_1,k_2,\ldots,k_{|\Omega|}\}$ is a set of $|\Omega|$ **winding numbers**.

- We focus on the **NOON state** ($|\Omega|=2$) and the **W state** ($|\Omega|=3$),

$$|\mathrm{NOON}\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle),$$



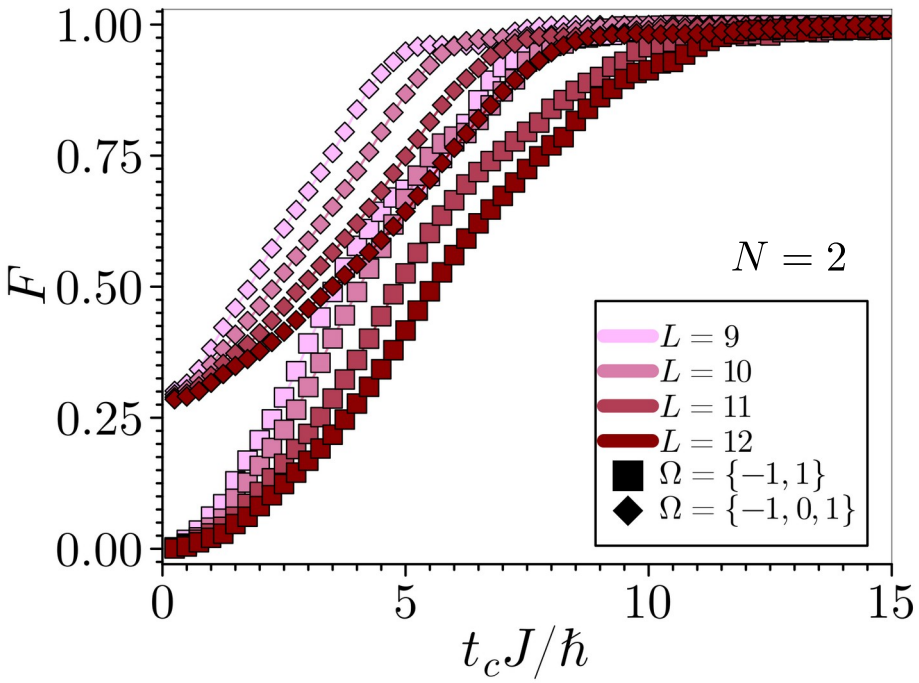
$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|N,0,0\rangle + |0,N,0\rangle + |0,0,N\rangle).$$



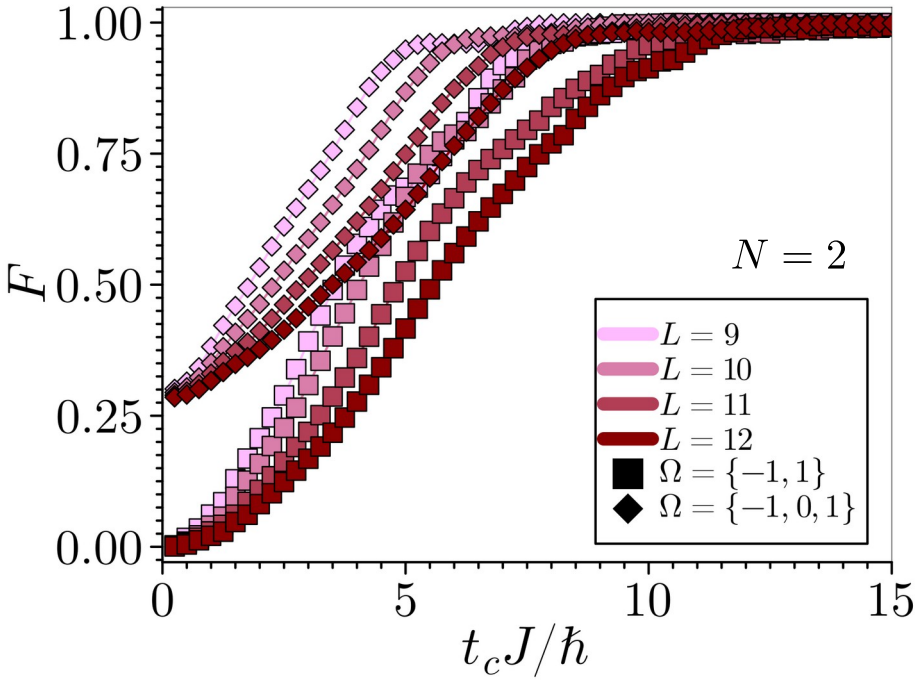
- We use the **GRAPE** algorithm.

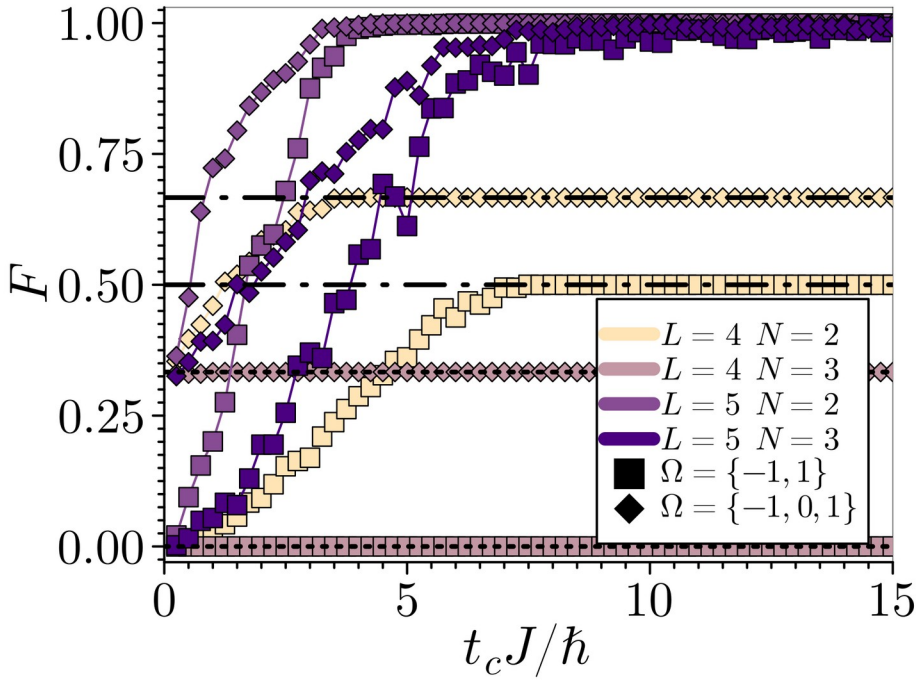
N. Khaneja *et al*., Journal of Magnetic Resonance **172**, 296 (2005). M. H. Goerz *et al*., Quantum **6**, 871 (2022),

Results


1.00
0.75
0.50
0.25
0.00
F
0
5
10
15
$t_c J/\hbar$
$N = 2$
$L = 9$
$L = 10$
$L = 11$
$L = 12$
$\Omega = \{-1, 1\}$
$\Omega = \{-1, 0, 1\}$

Results



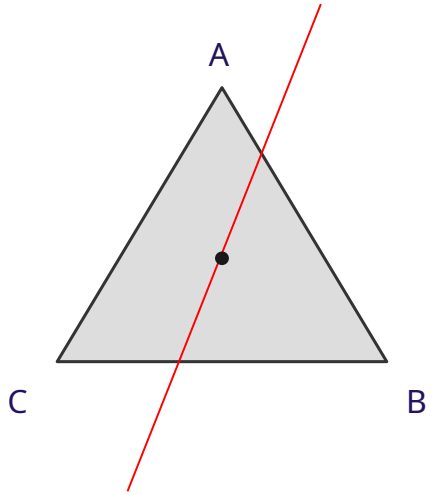


- **Full fidelity** can be achieved only for **selected combinations of the number of atoms and sites**.

Controllability

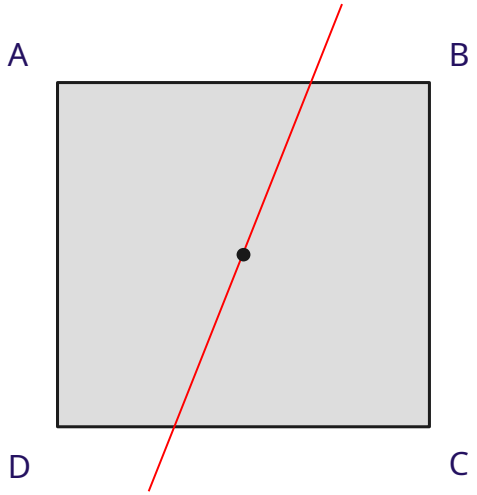
- **Symmetry constraints** provide **upper bounds** for the **fidelity**.

Odd L: No symmetry



Inversion symmetry:
change **r** by **-r** for each point.

Even L: Symmetry



- For an **even number of sites**, the **evolution** is **constrained** to the **initial subspace** (symmetric or antisymmetric).

  → Full fidelity cannot be achieved.

Conclusions

- **Quantum optimal control** can be used to prepare **entangled currents** in dipolar ring systems.
- However, **symmetries** impose **fundamental limitations** on the **fidelity**.
- Future work can include the consideration of **two-dimensional lattices** and **continuous rings**.

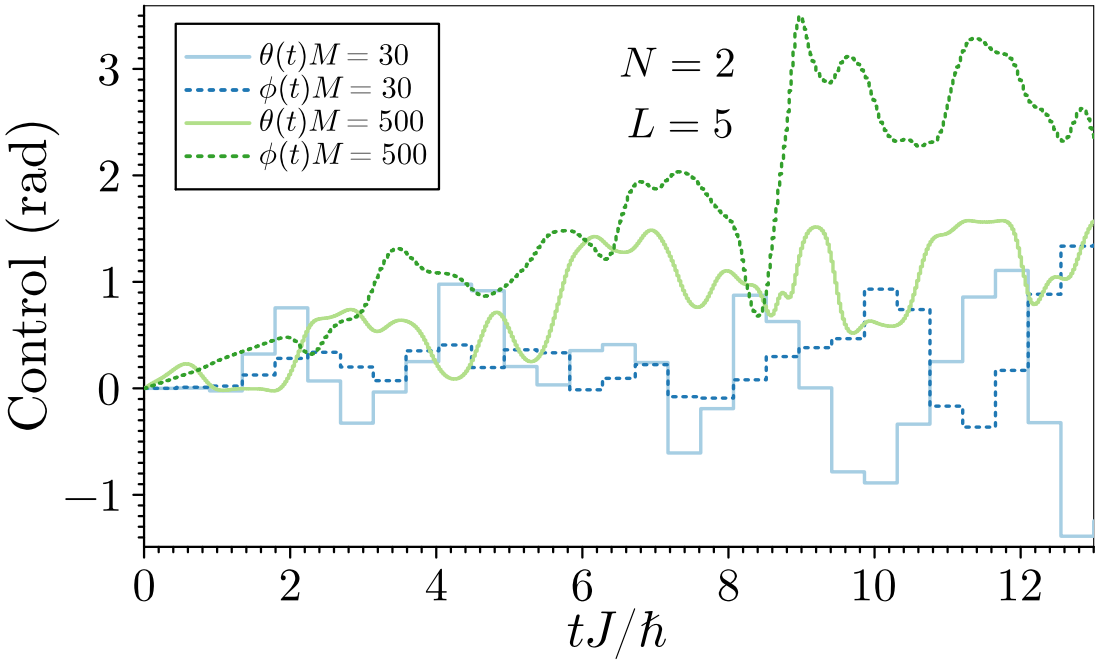
H. Briongos-Merino *et al*., Phys. Rev. Research **8**, 033121 (2026)

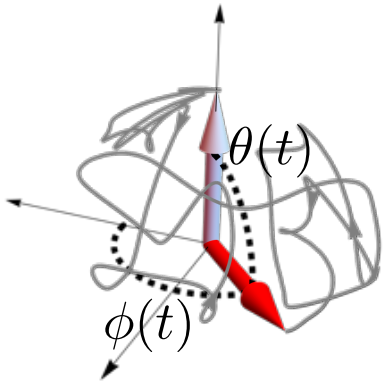
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Optimised trajectories





M : control steps

Controllability summary

		Symmetric Rings (even *L*)		**Non-symmetric Rings** (odd *L*)
		Even *k*	Odd *k*	
Even number of bosons	*N = 2*	Limited by the DI eigenstate		Completely Controllable
	N > 2	Reachable	Reachable	
Odd number of bosons		Reachable	Unreachable	

Dipolar immune (DI) bound:

$$F_{\max} = 1 - \left|\langle \Psi_{\mathrm{DI}} | \Psi_{\mathrm{EC}} \rangle\right|^2$$
$$= 1 - \left| \frac{1}{\sqrt{|\Omega| L}} \sum_{k \in \Omega} (\delta_{k,L/4} + \delta_{k,3L/4}) \right|^2$$

Symmetry bound:

$$F_{\max} = \sum_{\substack{k \in \Omega \\ kN \text{ even}}} \frac{1}{|\Omega|}$$

GRAPE convergence

