Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

EXCELENCIA MARÍA DE MAEZTU 04/2025-03/2031

Functional renormalisation group for quantum many-body systems

Lecture 3:

Functional renormalisation group for Bose and Fermi gases

Felipe Isaule

7th of July, 2026

felipeisaule.github.io

felipe.isaule@uc.cl

Lecture 3

1. The **FRG** for **Bose gases**. Ansatz within the **derivative expansion**.
2. Calculation of **observables**.
3. The **FRG** for **Fermi gases**.
4. **Other applications** and overview.

Lecture 3

1. The **FRG** for **Bose gases**. Ansatz within the **derivative expansion**.
2. Calculation of **observables**.
3. The **FRG** for **Fermi gases**.
4. **Other applications** and overview.

FRG for Bose gases

Functional renormalization for Bose-Einstein condensation

S. Floerchinger and C. Wetterich
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany
(Received 29 January 2008; published 8 May 2008)

We investigate Bose-Einstein condensation for interacting bosons at zero and nonzero temperature. Functional renormalization provides us with a consistent method to compute the effect of fluctuations beyond the Bogoliubov approximation. For three-dimensional dilute gases, we find an upper bound on the scattering length a which is of the order of the microphysical scale—typically the range of the van der Waals interaction. In contrast to fermions near the unitary bound, no strong interactions occur for bosons with approximately pointlike interactions, thus explaining the high quantitative reliability of perturbation theory for most quantities. For zero temperature we compute the quantum phase diagram for bosonic quasiparticles with a general dispersion relation, corresponding to an inverse microphysical propagator with terms linear and quadratic in the frequency. We compute the temperature dependence of the condensate and particle density n, and find for the critical temperature T_c a deviation from the free theory, $\Delta T_c/T_c = 2.1 a n^{1/3}$. For the sound velocity at zero temperature we find very good agreement with the Bogoliubov result, such that it may be used to determine the particle density accurately.

DOI: 10.1103/PhysRevA.77.053603 PACS number(s): 03.75.Hh, 05.30.Jp, 05.10.Cc

PHYSICAL REVIEW E **83**, 031120 (2011)

Infrared behavior in systems with a broken continuous symmetry: Classical O(N) model versus interacting bosons

N. Dupuis
Laboratoire de Physique Théorique de la Matière Condensée, CNRS–UMR 7600, Université Pierre et Marie Curie, 4 Place Jussieu, F-75252 Paris Cedex 05, France
(Received 22 November 2010; published 18 March 2011)

In systems with a spontaneously broken continuous symmetry, the perturbative loop expansion is plagued by infrared divergences due to the coupling between transverse and longitudinal fluctuations. As a result, the longitudinal susceptibility diverges and the self-energy becomes singular at low energy. We study the crossover from the high-energy Gaussian regime, where perturbation theory remains valid, to the low-energy Goldstone regime characterized by a diverging longitudinal susceptibility. We consider both the classical linear O (N) model and interacting bosons at zero temperature, using a variety of techniques: perturbation theory, hydrodynamic approach (i.e., for bosons, Popov's theory), large-N limit, and nonperturbative renormalization group. We emphasize the essential role of the Ginzburg momentum scale p_G, below which the perturbative approach breaks down. Even though the action of (nonrelativistic) bosons includes a first-order time derivative term, we find remarkable similarities in the weak-coupling limit between the classical O(N) model and interacting bosons at zero temperature.

S. Floerchinger and C. Wetterich, Phys. Rev. A **77**, 053603 (2008).

N. Dupuis, Phys. Rev. E **83**, 031120 (2011).

Weakly-interacting Bose gas

- In the first part of this lecture, we will study the full quantum **weakly interacting Bose gas**.
- Its **microscopic action** reads

$$\mathcal{S}[\phi] = \int_x \left[\phi^\dagger(x)\left(\partial_\tau - \frac{\nabla^2}{2M} - \mu\right)\phi(x) + \frac{g}{2}(\phi^\dagger(x)\phi(x))^2\right].$$

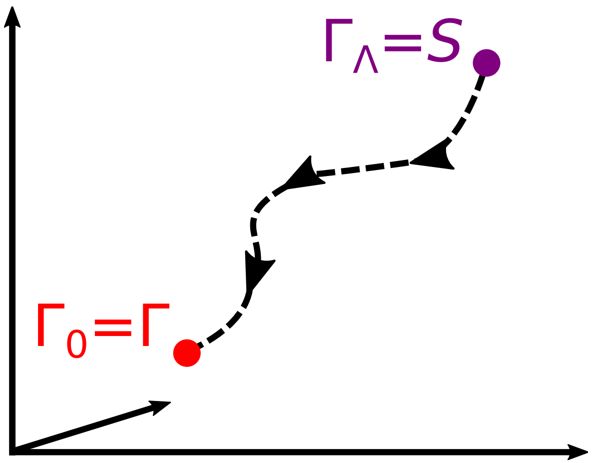
- We will connect the theory to the s-wave **scattering length**.
- Additionally, we will calculate **thermodynamic quantities**.

Functional renormalisation group for Bose gases

- We recall that our aim is to solve the **Wetterich equation**

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{tr}\left[\partial_k \mathcal{R}_k (\Gamma_k^{(2)} + \mathcal{R}_k)^{-1}\right], \qquad \Gamma^{(2)}_{k,\varphi_i\varphi_i} = \frac{\delta^2 \Gamma_k}{\delta\varphi_i(-\boldsymbol{q})\varphi_i(\boldsymbol{q})}$$

with $\Gamma_\Lambda = S$ as the **initial condition**.



- Because this equation cannot be solved exactly, one needs to propose a **truncated ansatz** for Γ_k.
- We will revise a truncation within the **derivative expansion**.

Ansatz for the effective action

- A common **truncated ansatz** for the **Bose gas** is

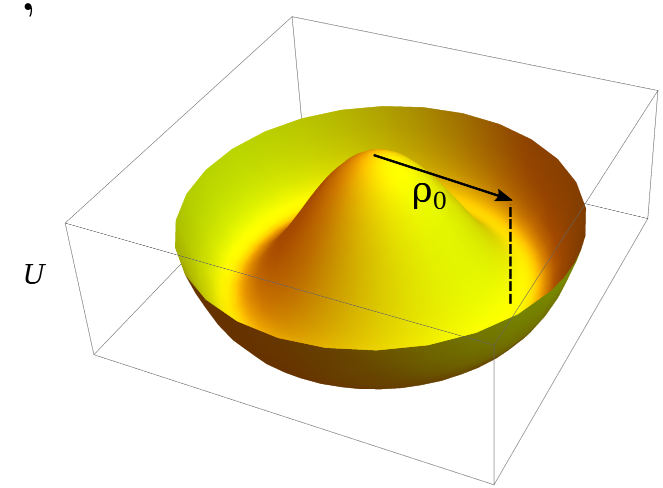
$$\Gamma_k[\boldsymbol{\varphi}] = \int_x \left[\varphi^\dagger(x)\left(S_k\partial_\tau - Z_k\frac{\nabla^2}{2M} - V_k\partial_\tau^2\right)\varphi(x) + U_k(\rho(x))\right],$$

where $\rho{=}\varphi^\dagger\varphi$, and

Classical fields $\boldsymbol{\varphi} = \langle\boldsymbol{\phi}\rangle$

$$U_k(\rho) = u_{0,k} + m_k^2(\rho - \rho_{0,k}) + \frac{\lambda_k}{2}(\rho - \rho_{0,k})^2,$$

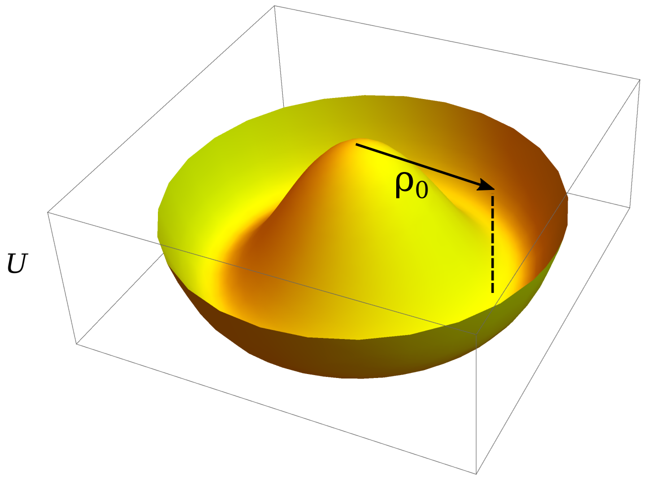
is the **effective potential**.



- The **couplings** Z_k, S_k, V_k, $u_{k,0}$, m_k^2, λ_k, and $\rho_{0,k}$ **depend on the scale** k.

Effective potential

- The **effective potential** plays the same role as in the O(2)-model.



$$\rho_0 = \varphi_0^2 \qquad \varphi(q) \longrightarrow \varphi_0 \delta(q)$$

Vacuum expectation value

- The **equilibrium** condition is again

$$\left.\frac{\partial U_k}{\partial \rho}\right|_{\rho=\rho_{0,k}} = 0.$$

- Therefore,

$$\rho_{0,k} > 0,\ m_k^2 = 0 \quad : \text{Symmetry is broken } (\textbf{broken phase})$$

$$\rho_{0,k} = 0,\ m_k^2 > 0 \quad : \text{Symmetry is not broken } (\textbf{symmetric phase})$$

- The **physical phase of the system** is dictated by the values of $\rho_{0,k}$ and m_k^2 in the **physical limit** $k\rightarrow 0$.

Real fields

- It is again convenient to introduce **real fields** φ_1 and φ_2 as follows

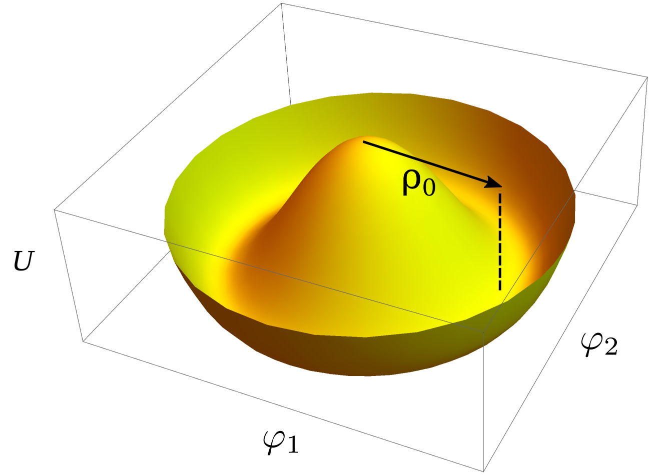
$$\varphi(x) = \frac{1}{2}(\varphi_1(x) + i\,\varphi_2(x)), \qquad \varphi^\dagger(x) = \frac{1}{2}(\varphi_1(x) - i\,\varphi_2(x)).$$

- We can impose that the **expectation value** is developed in the **real direction**.

- As with the O(2)-model, after taking the functional derivatives, we can evaluate the **background fields**

$$\varphi_1 = \sqrt{2\rho}, \qquad \varphi_2 = 0.$$

- Then, after taking the ρ-derivatives, we can evaluate at $\rho = \rho_0$.



Flow equations

- The **flow equations** are

$$\partial_k m_k^2 - \lambda_k \partial_k \rho_{0,k} = \left(\frac{\partial}{\partial \rho}\partial_k U_k\right)\Big|_{\rho=\rho_{0,k}},$$

$$\partial_k \lambda_k = \left(\frac{\partial^2}{\partial \rho^2}\partial_k U_k\right)\Big|_{\rho=\rho_{0,k}}.$$

$$\partial_k Z_k = \left(\frac{\partial}{\partial \boldsymbol{p}^2}\partial_k \Gamma^{(2)}_{k,\varphi_2\varphi_2}(p)\right)\Big|_{\rho=\rho_{0,k},p=0},$$

$$\partial_k S_k = \left(\frac{\partial}{\partial \nu_n}\partial_k \Gamma^{(2)}_{k,\varphi_2\varphi_1}(p)\right)\Big|_{\rho=\rho_{0,k},p=0},$$

$$\partial_k V_k = \left(\frac{\partial}{\partial \nu_n^2}\partial_k \Gamma^{(2)}_{k,\varphi_2\varphi_2}(p)\right)\Big|_{\rho=\rho_{0,k},p=0},$$

Driving terms

- Again, when $\rho_{0,k}>0$ we set $m_k^2=0$, and viceversa.

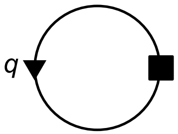
$p = (\nu_n, \boldsymbol{p})$: external momentum

Driving terms

- From the last lecture, we recall that the driving term of the **effective potential** is obtained from

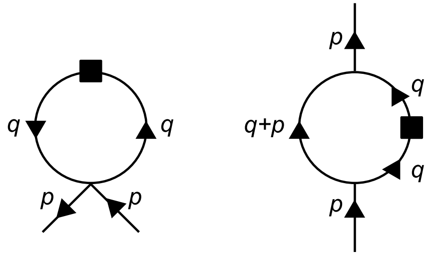
$G_k(q) = (\Gamma_k^{(2)}(q) + \mathcal{R}_k(q))^{-1}$

$$\partial_k U_k = \frac{1}{2}\mathrm{tr}\left[\partial_k \mathcal{R}_k(q) G_k(q)\right],$$



while that of the **two-point function** from

$$\partial_k \Gamma^{(2)}_{k,\phi_i\phi_j}(p) = -\frac{1}{2}\mathrm{tr}\left[\partial_k \mathcal{R}_k(q) G_k(q) \Gamma^{(4)}_{k,\phi_i\phi_j}(p,-p,q,-q) G_k(q)\right]$$
$$+\mathrm{tr}\left[\partial_k \mathcal{R}_k(q) G_k(q) \Gamma^{(3)}_{k,\phi_i}(p,q,-q-p) G_k(q+p) \Gamma^{(3)}_{k,\phi_j}(-p,q+p,-q) G_k(q)\right].$$



Driving terms

- The **two-point function** evaluated at the **background fields**

$$\Gamma_k^{(2)}(q) = \begin{pmatrix} V_k\omega_n^2 + \frac{Z_k}{2M}\boldsymbol{q}^2 + m_k^2 + \lambda_k(3\rho - \rho_{0,k}) & -S_k\omega_n \\ S_k\omega_n & V_k\omega_n^2 + \frac{Z_k}{2M}\boldsymbol{q}^2 + m_k^2 + \lambda_k(\rho - \rho_{0,k}) \end{pmatrix}.$$

where $q=(\omega_n,\boldsymbol{q})$.

- The **regulator matrix** and **propagator** are then

$$\mathcal{R}_k(q) = \begin{pmatrix} R_k(q) & 0 \\ 0 & R_k(q) \end{pmatrix} \qquad \longrightarrow \qquad G_k(q) = (\Gamma_k^{(2)}(q) + \mathcal{R}_k(q))^{-1}.$$

- In turn, the **vertices** are calculated from

$$\Gamma^{(3)}_{k,\varphi_i} = \frac{\delta\Gamma_k^{(2)}}{\delta\varphi_i}, \qquad \Gamma^{(4)}_{k,\varphi_i\varphi_j} = \frac{\delta\Gamma_k^{(2)}}{\delta\varphi_j\delta\varphi_i}.$$

Regulator

- In principle, one should use a **regulator** that **regulates both momenta and frequency**.
- However, unless one requires high accuracy, one can use a **frequency-independent regulator**.
- As seen in the past lectures, examples include the **exponential regulator**,

$$R_k(\boldsymbol{q}) = \frac{Z_k}{2M} \frac{\boldsymbol{q}^2}{\exp(\boldsymbol{q}^2/k^2) - 1},$$

and the **optimised Litim regulator**,

$$R_k(\boldsymbol{q}) = \frac{Z_k}{2M}(k^2 - \boldsymbol{q}^2)\Theta(k^2 - \boldsymbol{q}^2).$$

Θ : Heaviside step function

Propagator

- For **frequency-independent regulators** $R_k(q)\!=\!R_k(\boldsymbol{q})$, we can write the **propagator** as

$$G_k(q) = \frac{1}{\det G^{-1}(q;\rho)} \begin{pmatrix} E_{2,k}(\boldsymbol{q};\rho) + V_k\omega_n^2 & S_k\omega_n \\ -S_k\omega_n & V_k\omega_n^2 + E_{1,k}(\boldsymbol{q};\rho) + V_k\omega_n^2) \end{pmatrix}.$$

where the **regulated energies** are again

$$E_{1,k}(\boldsymbol{q};\rho) = \frac{Z_k}{2M}\boldsymbol{q}^2 + m_k^2 + \lambda_k(3\rho - \rho_{0,k}) + R_k(\boldsymbol{q}),$$

$$E_{2,k}(\boldsymbol{q};\rho) = \frac{Z_k}{2M}\boldsymbol{q}^2 + m_k^2 + \lambda_k(\rho - \rho_{0,k}) + R_k(\boldsymbol{q}),$$

and

$$\det G^{-1}(q;\rho) = S_k^2\omega_n^2 + (E_{1,k}(\boldsymbol{q};\rho) + V_k\omega_n^2)(E_{2,k}(\boldsymbol{q};\rho) + V_k\omega_n^2).$$

Driving terms

- The **driving term** for the **effective potential** is

$$\partial_k U_k = \int_q \frac{E_{1,k}(\boldsymbol{q};\rho) + E_{2,k}(\boldsymbol{q};\rho) + 2V_k\omega_n^2}{2\det G^{-1}(q;\rho)} \partial_k R_k(\boldsymbol{q}),$$

where

$$\int_q = T\sum_{n=-\infty}^{\infty}\int_{\boldsymbol{q}}, \qquad \int_{\boldsymbol{q}} = \frac{1}{(2\pi)^d} S_d \int_0^\infty q^{d-1}dq.$$

- We can solve the **frequency integral** analytically for any **frequency-independent regulator**.
- We do this **before solving the flow.**

Frequency integration

- To perform the **frequency integration**, we must examine the **poles**.
- The **denominators** of the **driving term** will always be composed of $\det {G_k}^{-1}$.
- Thus, the **poles** are obtained from

$$\det G_k^{-1}(q) = 0.$$

- Therefore, these are

$$q_0^2 = \frac{1}{2V_k}\Bigg(S_k^2 + V_k(E_{1,k}(\boldsymbol{q}) + E_{2,k}(\boldsymbol{q})) \\ \pm\sqrt{(S_k^2 + V_k(E_{1,k}(\boldsymbol{q}) + E_{2,k}(\boldsymbol{q}))^2 - 4V_k^2 E_{1,k}(\boldsymbol{q})E_{2,k}(\boldsymbol{q})}\Bigg).$$

where $q_0 = i\,\omega_n$.

Frequency integration (T=0)

- At **zero temperature**, we can perform the **Cauchy integral**.

$$T\sum_n \xrightarrow{T=0} -\frac{i}{2\pi}\int_{-\infty}^{\infty} dq_0. \qquad \omega_n \to -iq_0$$

- Thus, we calculate the **residues** from the **poles** in one half of the complex plane.

- For V_k=0, the **driving term** for the **effective potential** becomes

$$\partial_k U_k = \frac{1}{4S_k}\int_{\boldsymbol{q}} \frac{E_{1,k}(\boldsymbol{q};\rho)+E_{2,k}(\boldsymbol{q};\rho)}{\sqrt{E_{1,k}(\boldsymbol{q};\rho)E_{2,k}(\boldsymbol{q};\rho)}}\partial_k R_k(\boldsymbol{q}).$$

Frequency integration (T>0)

- At **finite temperatures**, we must perform the **Matsubara sum**.

$$T\sum_n ,$$

with $\omega_n = 2\pi n T$.

- These can also be solved through **residues**.

G. D. Mahan, Gerald D. Many-particle physics (Springer Science & Business Media, 2013). A. Nieto, Computer physics communications **92**, 54 (1995).

- For $V_k = 0$, the **driving term** for the **effective potential** becomes

$$\partial_k U_k = \frac{1}{4S_k}\int_{\boldsymbol{q}} \frac{E_{1,k}(\boldsymbol{q};\rho)+E_{2,k}(\boldsymbol{q};\rho)}{\sqrt{E_{1,k}(\boldsymbol{q};\rho)E_{2,k}(\boldsymbol{q};\rho)}} \coth\left(\frac{\sqrt{E_{1,k}(\boldsymbol{q};\rho)E_{2,k}(\boldsymbol{q};\rho)}}{2TS_k}\right)\partial_k R_k(\boldsymbol{q}).$$

- Note that for $T \to 0$ we recover the zero-temperature driving term.

Momentum integration

- After doing the **frequency integrals**, we are left with simpler **momentum integrals**, as seen in the **past lecture**.
- In general, we need to deal with **integro-differential equations**.
- For the employed ansatz, the $\boldsymbol{q}$-integrals can be integrated analytically with the **Litim regulator**. These are of the type

$$\int_{\boldsymbol{q}} \quad \longrightarrow \quad \int_{|\boldsymbol{q}|<k} = \frac{S_d}{(2\pi)^d} \int_0^k q^{d-1} dq = \frac{S_d}{(2\pi)^d} \frac{k^d}{d},$$

where $S_d = 2\pi^{d/2}/\Gamma(d/2)$.

→ Differential equations instead of integro-differential ones.

Initial conditions

- For the initial conditions, we set

$$\Gamma_\Lambda[\varphi] = \mathcal{S}[\varphi].$$

- For $\mu > 0$, we obtain the following **initial conditions**

$$m_\Lambda^2 = 0, \qquad \rho_{0,\Lambda} = \mu/g, \qquad \lambda_\Lambda = g,$$

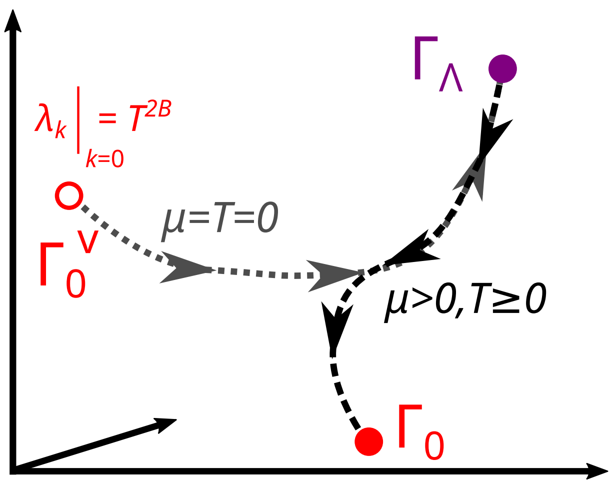
$$Z_\Lambda = S_\Lambda = 1, \qquad V_\Lambda = 0.$$

- Therefore, the **flow starts** in the **broken phase**.

Initial conditions (two-body scattering)

- To **connect** with **physical scattering**, we **renormalise** the **interaction coupling** λ_k.
- We impose that the flow in **vacuum** $(\mu{=}0, T{=}0)$ recovers the **known two-body physics**.

$$\lambda_{k\to 0}^{(\text{vacuum})} = T^{(2\text{B})}.$$



- In **three dimensions**, this is

$$T^{(2\text{B})} = \frac{4\pi a}{M},$$

where a is the ***s*-wave scattering length**.

Initial conditions (two-body scattering)

- The **flow** of λ_k in **vacuum** $(\mu=0, T=0)$ is dictated by

$$\partial_k \lambda_k = \frac{\lambda^2}{2} \int_{\boldsymbol{q}} \frac{\partial_k R_k}{E_{s,k}^2(\boldsymbol{q})},$$

where

$$E_{s,k}(\boldsymbol{q}) = E_{1,k}(\boldsymbol{q}; \rho = 0) = E_{2,k}(\boldsymbol{q}; \rho = 0) = \frac{\boldsymbol{q}^2}{2M} + R_k(\boldsymbol{q}).$$

- We note that in vacuum $Z_k=1$ for all k.

- By noting that $\partial_k E_{s,k}=\partial_k R_k$, we can integrate between $k=0$ and $k=\Lambda$. We obtain

$$\frac{1}{\lambda_\Lambda} - \frac{1}{\lambda_0} = \frac{1}{2} \int_{\boldsymbol{q}} \left[\frac{1}{E_{s,\Lambda}(\boldsymbol{q})} - \frac{1}{E_{s,0}(\boldsymbol{q})} \right].$$

Initial conditions (two-body scattering)

- By imposing that $\lambda_0 = T^{(2B)}$ and using the **Litim regulator**, for **three dimensions** we obtain

$$\lambda_\Lambda = \left(\frac{M}{4\pi a} - \frac{M}{3\pi^2}\Lambda \right)^{-1}.$$

- While this dictates the flow of λ_k in vacuum, we use it as its **initial condition** in-medium.

- Note that λ_Λ diverges at $\Lambda = 3\pi/(4a)$.

- This is a **constraint** of **repulsive interactions**, as these can be seen as **hard spheres**.

- More generally, in **three dimensions** the flow should be restricted to

$$\Lambda < a^{-1}.$$

Hard-sphere radius

Initial scale

- Nevertheless, it is important to **start the flow above** the **relevant scales** of the system

$$\Lambda \gg p_h,\, \lambda_{\text{th}}^{-1},$$

where

$$p_h = \sqrt{2M\mu},$$

Healing scale

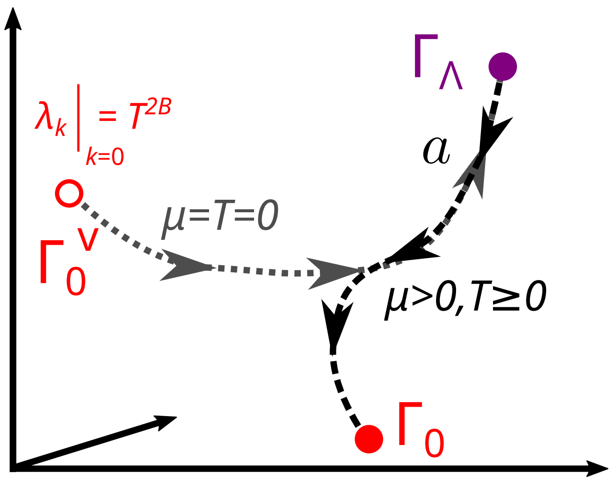
$$\lambda_{\text{th}} = \sqrt{\frac{2\pi}{MT}}.$$

Thermal scale

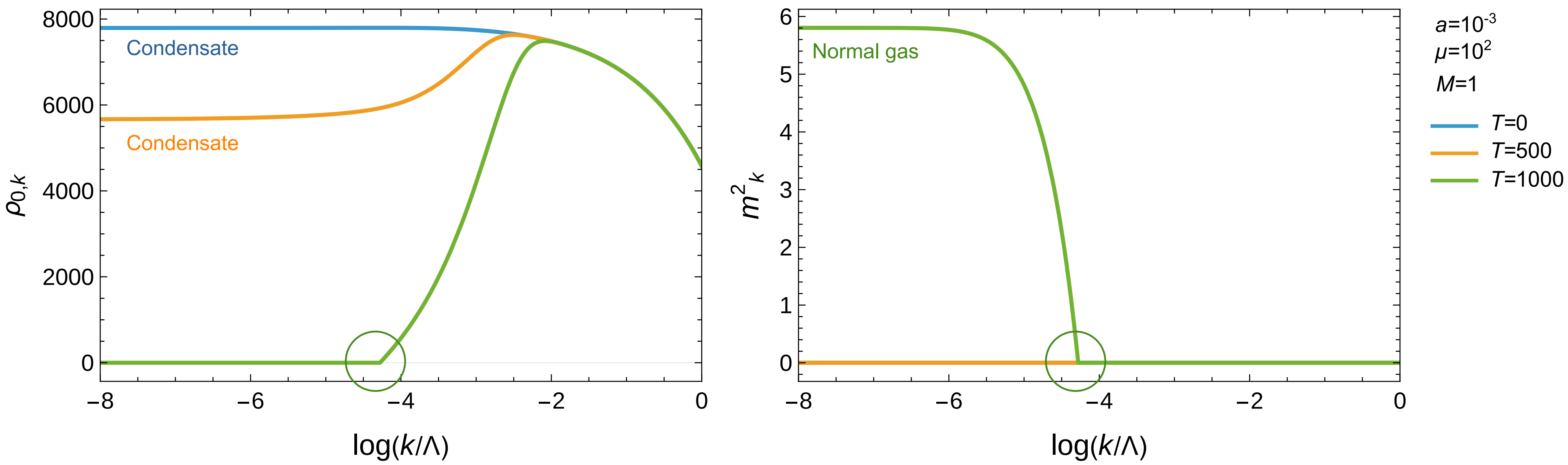
- Combined with the previous upper bound set by the scattering length, the flow is restricted to **weak interactions**.

Physical inputs

- In summary, the **physical inputs** of the **RG flow** are μ, a, and T.
- The **chemical potential** and **scattering lengths** are incorporated through the **initial conditions**.
- The **temperature** is incorporated into the **Matsubara sums**.

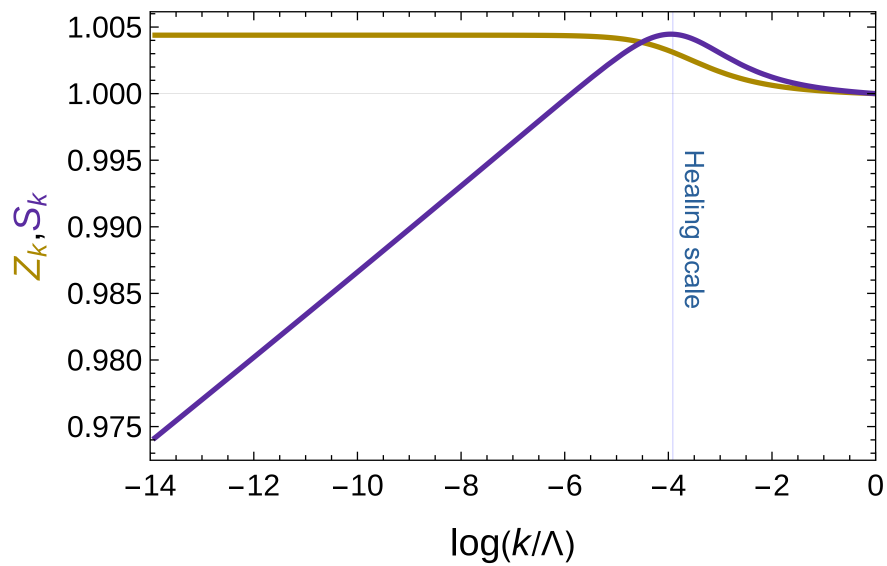


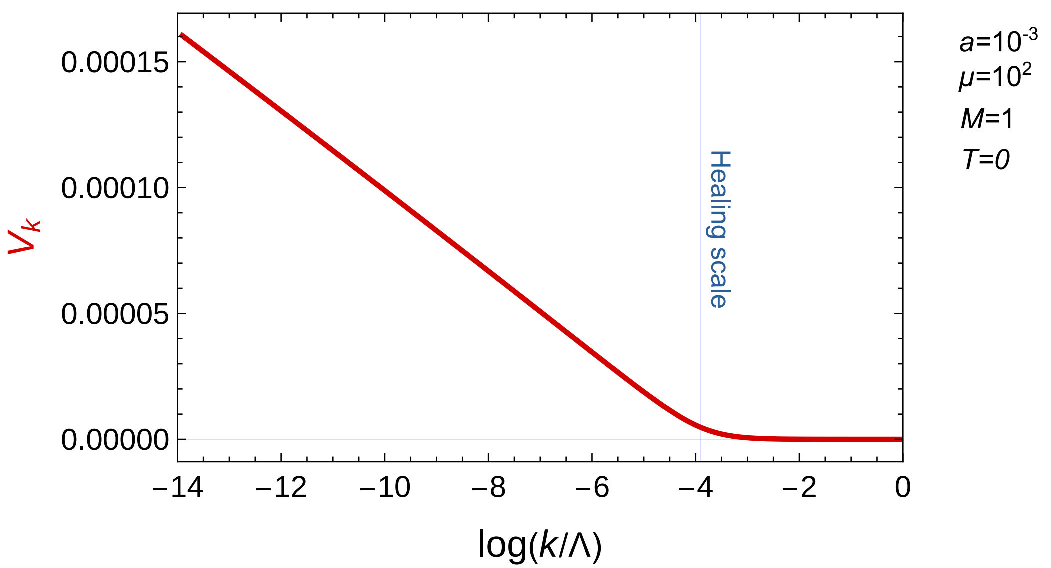
Flow examples (d=3)



- The **blue** and **orange** lines show flows for **condensed (broken) systems**.
- The **green** lines show a flow for a **normal (symmetric) system**.

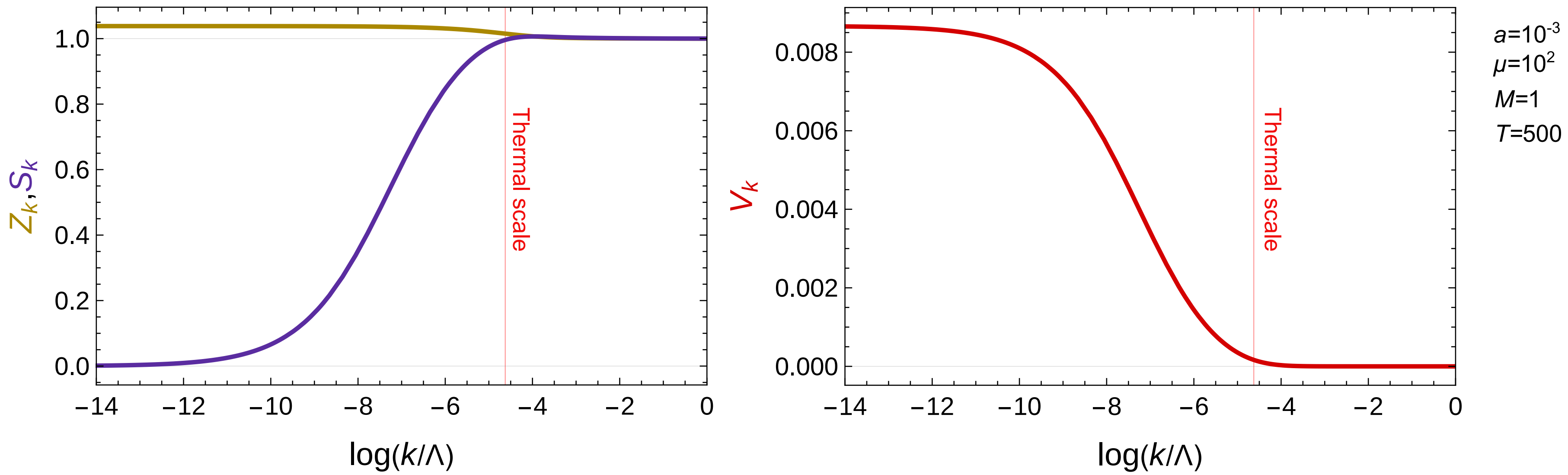
Flow examples (d=3)





- At **zero temperature**, Z_k converges to a **finite value** for $k \to 0$.
- S_k **vanishes** for small k. However, it vanishes very slowly (logarithmically).
- In turn, V_k is **generated** (slowly) during the flow.
- Note that the changes (from the microscopic values) occur around the **healing scale**.

Flow examples (d=3)



- At **finite temperatures**, Z_k also converges to a **finite value** for $k \to 0$.
- S_k **vanishes** rapidly small k. In turn, V_k is also **generated** (rapidly) during the flow.
- Note that the initial changes (from the microscopic values) occur around the thermal scale.

Behaviour of the renormalisation factors

- S_k vanishes for $k \to 0$ due to **Ward identities**. More specifically, the **anomalous self-energy** must **vanish**.
  A. A. Nepomnyashchy and Yu. A. Nepomnyashchy, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 3 (1975).
  - → The term with V_k must be **generated** during the flow.
- Nevertheless, at **zero temperature** S_k vanishes logarithmically, and the observables converge before V_k becomes important.
  - → We can neglect V_k at $d=3$ and $T=0$.
- **In other cases**, V_k is **important** for obtaining good accuracy and stabilising the flows.

Lecture 3

1. The **FRG** for **Bose gases**. Ansatz within the **derivative expansion**.
2. Calculation of **observables**.
3. The **FRG** for **Fermi gases**.
4. **Other applications** and overview.

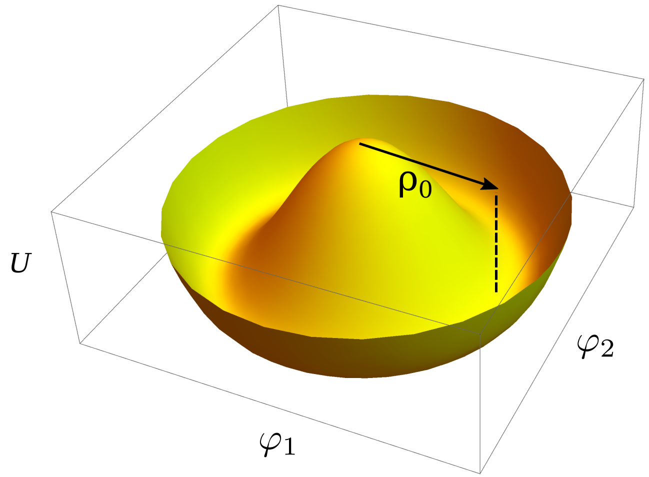
Condensate and superfluid density

- The **order parameter** $\rho_{0,k}$ is also the scale-dependent **condensate density** of the gas.
- Additionally, the **superfluid density** (stiffness) is given by

N. Dupuis, Phys. Rev. A **80**, 043627 (2009).

$$\rho_s = Z_k \rho_{0,k}.$$

- We obtain their **physical values** from their values for $k \to 0$.



Grand-canonical potential

- We recall that the **grand-canonical potential** is related to the **effective action** as

$$\Omega = \beta^{-1}\Gamma[\boldsymbol{\varphi}_0],$$

where

$$d\Omega = -Pd\mathcal{V} - SdT - Nd\mu.$$

- Therefore, we can extract the **thermodynamics** from $U_{k\to 0}$.

Number density

- The **physical** (number) **density** is given by

$$n = -\frac{\partial U_{k\to 0}}{d\mu}\bigg|_{\rho=\rho_0}.$$

Number of particles per volume

- Thus, one could calculate $u_{k\to 0}$ for several values of μ, and then take a numerical derivative.

- Instead, it is more convenient to define a **scale-dependent density** n_k and extract its physical value at $k\to 0$.

- We modify the **effective potential** to

S. Floerchinger and C. Wetterich, Phys. Rev. A 77, 053603 (2008).

$$U_k(\rho) = u_{0,k} + m_k^2(\rho - \rho_{0,k}) + \frac{\lambda_k}{2}(\rho - \rho_{0,k})^2,$$
$$-\left(n_k + n_{1,k}(\rho - \rho_{0,k}) + \frac{n_{2,k}}{2}(\rho - \rho_{0,k})^2\right)\delta\mu,$$

where $\delta\mu$ is an artificial shift to the chemical potential.

Number density (flow equations)

- The **flow equations** of the new couplings are

$$
\begin{aligned}
\partial_k n_k - n_{1,k}\partial_k \rho_{0,k} &= \frac{\partial}{\partial(\delta\mu)}\left(\partial_k U_k\right)\Big|_{\rho=\rho_{0,k},\delta\mu=0},\\
\partial_k n_{1,k} - n_{2,k}\partial_k \rho_{0,k} &= \frac{\partial}{\partial(\delta\mu)}\left(\frac{\partial}{\partial\rho}\partial_k U_k\right)\Big|_{\rho=\rho_{0,k},\delta\mu=0},\\
\partial_k n_{2,k} &= \frac{\partial}{\partial(\delta\mu)}\left(\frac{\partial^2}{\partial\rho^2}\partial_k U_k\right)\Big|_{\rho=\rho_{0,k},\delta\mu=0}.
\end{aligned}
$$

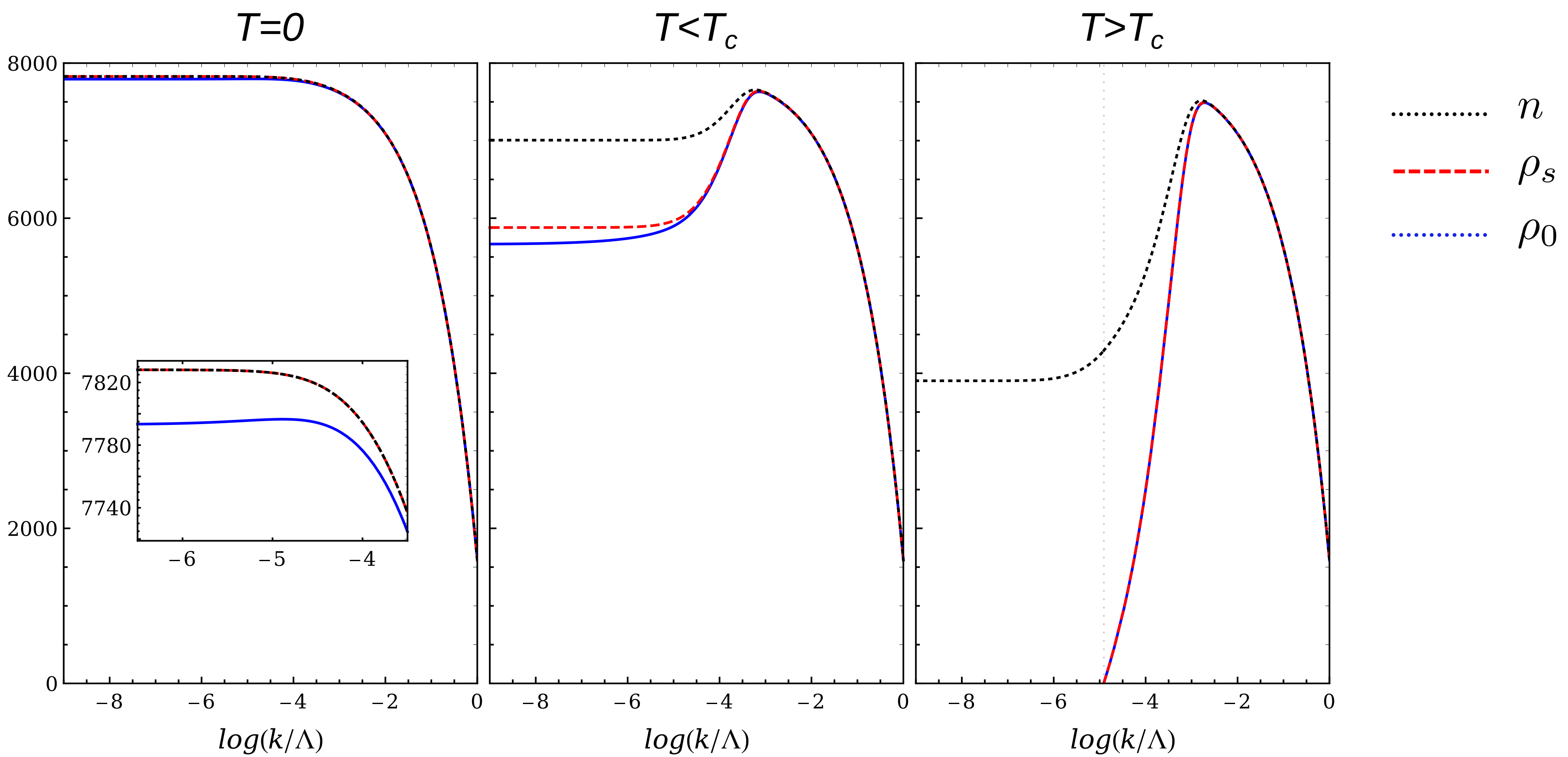
Driving terms

- Their **initial conditions** are

$$
u0,\Lambda = \rho_{0,\Lambda}, \qquad u_{1,\Lambda} = 1, \qquad u_{2,\Lambda} = 0.
$$

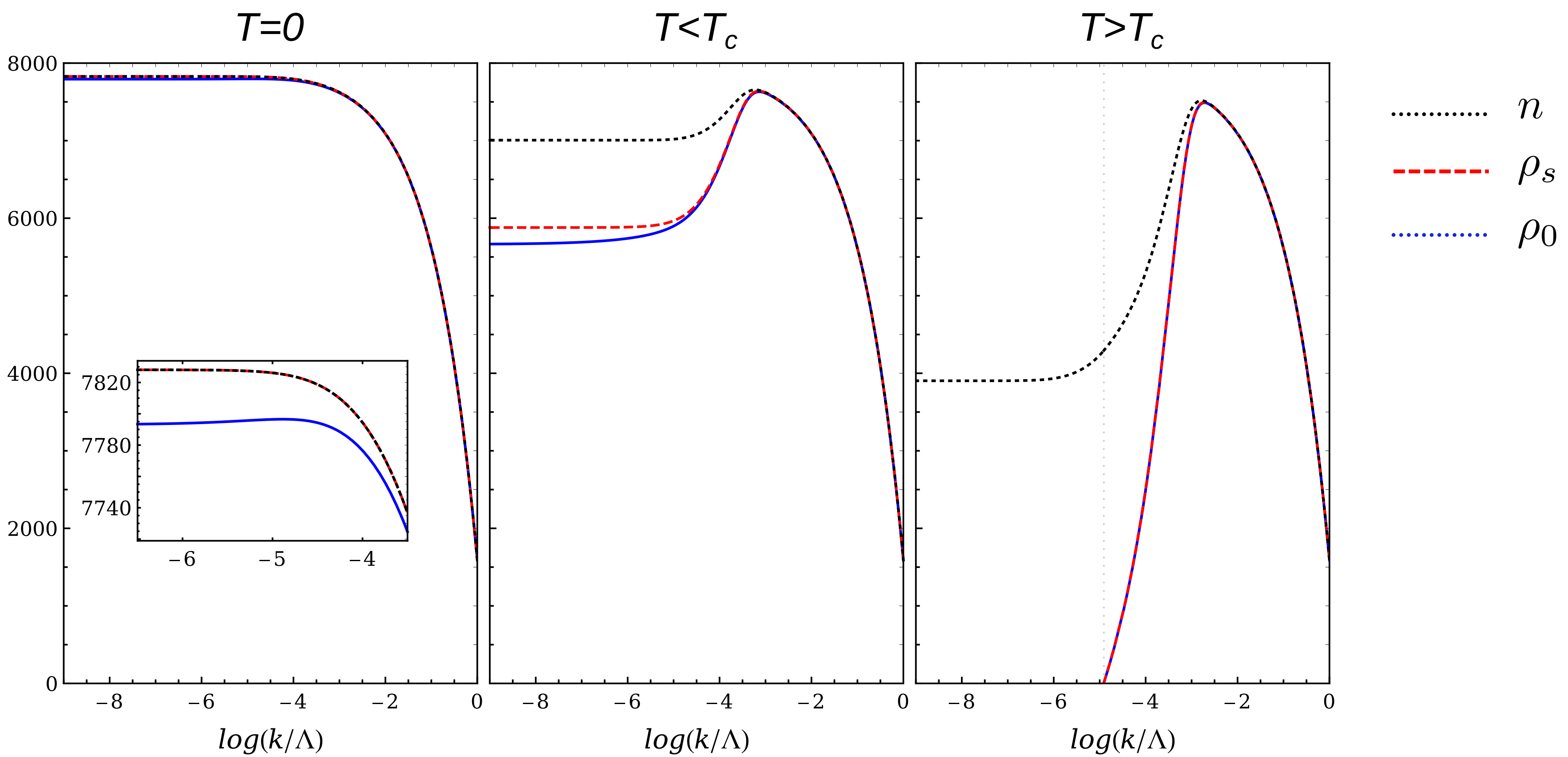
- Note that we must set $\delta\mu=0$ in all the previous flow equations.

Flow examples (d=3)



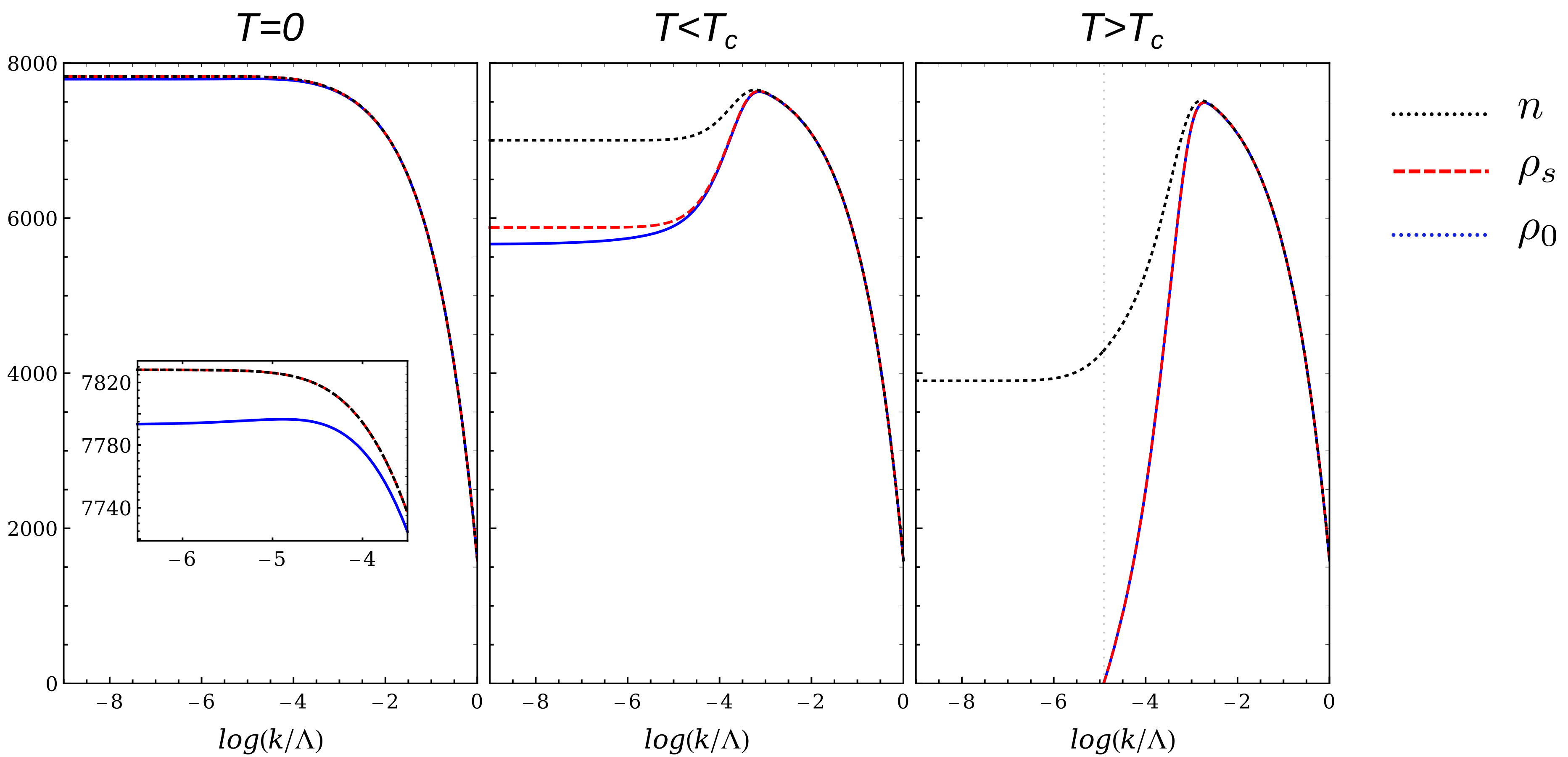
- At **zero temperature** we correctly obtain that $n = \rho_s$.
  J. Gavoret and P. Nozi`eres, Ann. Phys. (NY) **28**, 349 (1964).
- We observe a small **condensate depletion** $\rho_0 < n$.

Flow examples (d=3)



- At **finite temperatures** we observe that $n < \rho_s < \rho_0$.
- There is a larger **condensate depletion**.

Flow examples (*d*=3)



- In the **normal phase** we observe that $\rho_s{=}\rho_0{=}0$, but $n{>}0$, as expected.

Entropy

- The **physical entropy density** is given by

$$s = -\frac{\partial U_{k\to 0}}{dT}\bigg|_{\rho=\rho_0}.$$

Entropy per volume

- Thus, we could calculate $u_{k\to 0}$ for several values of T, and then take a numerical derivative.
- Again, it is more convenient to define a **scale-dependent entropy density** s_k and extract its physical value at $k \to 0$.
- We do not need to modify the effective potential, as the T -derivative is taken directly over the Matsubara sums.
- The **flow equation** reads

$$\partial_k s_k = \frac{\partial}{\partial T}\left(\partial_k U_k\right)\bigg|_{\rho=\rho_{0,k}}.$$

Pressure

- Finally, the **pressure** is given by

$$P = -U_{k\to 0}(\rho_0) = -u_{0,k\to 0}.$$

← Zero-point

- However, this **direct extraction** of the pressure is often **not reliable** within the employed approximations.
- This is because the calculation of the zero-point function requires the delicate cancellation of **counterterms**.

J.-P. Blaizot, A. Ipp, R. Méndez-Galain, N. Wschebor, Nuclear Phys. A **784**, 376 (2007).

- Note that the derivatives do not present this problem.

Pressure

- We can calculate the **pressure** from the **physical values** of the **density** and **entropy**.

- From the **Maxwell relations**, we have that

$$n = \frac{\partial P}{\partial \mu}\bigg|_{T,\mathcal{V}}, \qquad s = \frac{\partial P}{\partial T}\bigg|_{\mu,\mathcal{V}}.$$

- Thus, the **zero-temperature pressure** is obtained from integrating the **density** over the **chemical potential**

$$P(\mu, T=0) = \int_0^{\mu} n_{k\to 0}(\mu')d\mu'.$$

- Note that we have used that the pressure is zero in the vacuum $P(\mu{=}0, T{=}0){=}0$.

Pressure

- Afterwards, we can calculate the **finite-temperature** **pressure** by integrating the **entropy** over the **temperature**

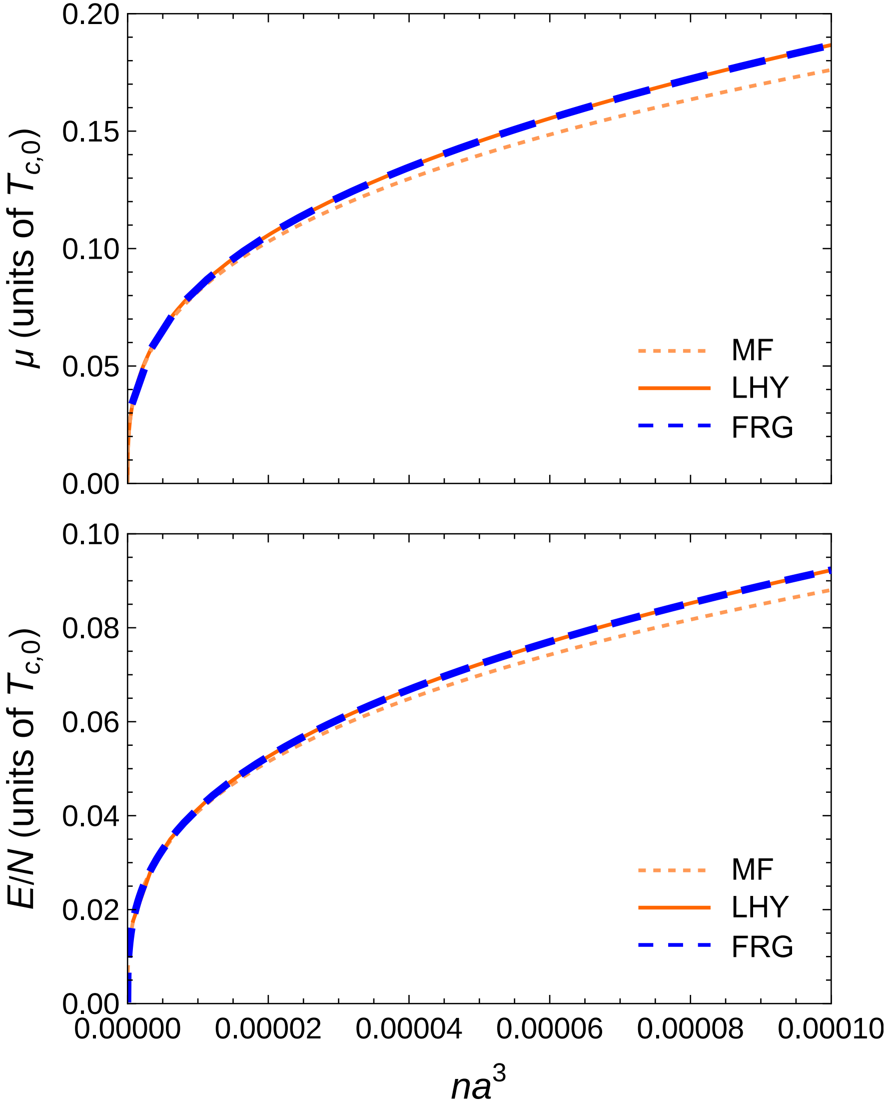
$$P(\mu, T) = P(\mu, T=0) + \int_0^T s_{k\to 0}(T')dT'.$$

- The **energy density** is then obtained from

$$\epsilon = -P + n\mu + sT.$$

- The energy per particle is then $E/N{=}\epsilon/n$.

Example (d=3)



- We obtain perfect **agreement** with the **LHY corrections**,

$$\mu = \frac{4\pi a n}{M}\left(1 + \frac{32}{3\pi^{1/2}}(na^3)^{1/2}\right),$$

$$E/N = \frac{2\pi a n}{M}\left(1 + \frac{128}{15\pi^{1/2}}(na^3)^{1/2}\right).$$

FRG for Bose gases

- The FRG is able to describe Bose gases by correctly **taking fluctuations into account**.
- The **low-temperature** behaviour of Bose gases is already **well described by perturbative approaches**.
- However, the FRG enables us to easily examine higher temperatures and criticality.

Lecture 3

1. The **FRG** for **Bose gases**. Ansatz within the **derivative expansion**.
2. Calculation of **observables**.
3. The **FRG** for **Fermi gases**.
4. **Other applications** and overview.

FRG for Fermi gases

Ann. Phys. (Berlin) **522**, No. 9, 615 – 656 (2010) / **DOI** 10.1002/andp.201010458

Functional renormalization group approach to the BCS-BEC crossover

Sebastian Diehl[1,*], **Stefan Floerchinger**[2], **Holger Gies**[3], **Jan Martin Pawlowski**[2], and **Christof Wetterich**[2]

[1] Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, Otto-Hittmair-Platz 1, 6020 Innsbruck, Austria
[2] Institut für Theoretische Physik, Philosophenweg 16, 69120 Heidelberg, Germany
[3] Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität, Max-Wien-Platz 1, 07743 Jena, Germany

Received 5 February 2010, accepted 2 June 2010 by A. Wipf
Published online 19 July 2010

Key words Strongly correlated fermions, phase transitions, condensation phenomena.

The phase transition to superfluidity and the BCS-BEC crossover for an ultracold gas of fermionic atoms is discussed within a functional renormalization group approach. Non-perturbative flow equations, based on an exact renormalization group equation, describe the scale dependence of the flowing or average action. They interpolate continuously from the microphysics at atomic or molecular distance scales to the macroscopic physics at much larger length scales, as given by the interparticle distance, the correlation length, or the size of the experimental probe. We discuss the phase diagram as a function of the scattering length and the temperature and compute the gap, the correlation length and the scattering length for molecules. Close to the critical temperature, we find the expected universal behavior. Our approach allows for a description of the few-body physics (scattering and molecular binding) and the many-body physics within the same formalism.

REVIEW

Functional renormalization for the Bardeen–Cooper–Schrieffer to Bose–Einstein condensation crossover

BY MICHAEL M. SCHERER[1], STEFAN FLOERCHINGER[2] AND HOLGER GIES[1,*]

[1] *Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, Max-Wien-Platz 1, 07749 Jena, Germany*
[2] *Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*

We review the functional renormalization group (RG) approach to the Bardeen–Cooper–Schrieffer to Bose–Einstein condensation (BCS–BEC) crossover for an ultracold gas of fermionic atoms. Formulated in terms of a scale-dependent effective action, the functional RG interpolates continuously between the atomic or molecular microphysics and the macroscopic physics on large length scales. We concentrate on the discussion of the phase diagram as a function of the scattering length and the temperature, which is a paradigm example for the non-perturbative power of the functional RG. A systematic derivative expansion provides for both a description of the many-body physics and its expected universal features as well as an accurate account of the few-body physics and the associated BEC and BCS limits.

Keywords: functional renormalization group; ultracold fermionic atoms; Bardeen–Cooper–Schrieffer to Bose–Einstein condensation crossover

S. Diehl, S. Floerchinger, H. Gies, J. M. Pawlowski, and C. Wetterich, Ann. Phys. (Berlin) **522**, 615 (2010).

M. M. Scherer, S. Floerchinger, and H. Gies, Phil. Trans. R. Soc. A **369**, 2779 (2011).

FRG for fermionic systems

- In **fermionic** systems, one works with **Grassmann fields** with **anticommutative** properties.
- The **Wetterich equation** takes the form

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{str} \left[\partial_k \mathcal{R}_k (\Gamma_k^{(2)} + \mathcal{R}_k)^{-1} \right],$$

where the **supertrace** is a trace that adds a **minus sign** to the **fermionic** elements

$$\mathrm{str}(A) = \mathrm{tr}(A_{\mathrm{bos}}) - \mathrm{tr}(A_{\mathrm{fer}}).$$

FRG for fermionic systems

- The **functional derivatives** are **alternating** **derivatives** from the **left and the right**,

$$\Gamma_k^{(2)} = \left(\frac{\delta\Gamma_k}{\delta\varphi}\right)\frac{\overleftarrow{\delta}}{\delta\varphi}, \qquad \Gamma_k^{(3)} = \frac{\delta}{\delta\varphi}\left(\left(\frac{\delta\Gamma_k}{\delta\varphi}\right)\frac{\overleftarrow{\delta}}{\delta\varphi}\right), \qquad \ldots$$

- The integrals retain their form

$$\int_q = T\sum_{n=-\infty}^{\infty}\int_{\boldsymbol{q}}, \qquad \int_{\boldsymbol{q}} = \frac{1}{(2\pi)^d}S_d\int_0^{\infty} q^2 dq,$$

where the **fermionic Matsubara frequencies** are $\omega_n = 2\pi(n+1/2)T$.

Fermi gas

- A **balanced two-component Fermi gas** is described by

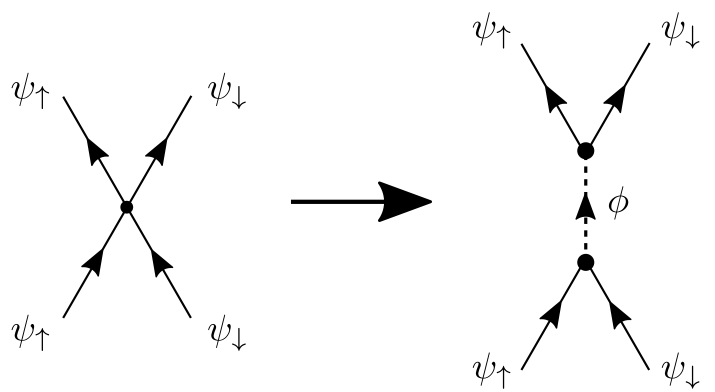
$$\mathcal{S} = \int_x \left[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2M} - \mu \right) \psi_\sigma + g\psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \right],$$

ψ_σ : fermionic fields

where is the strength of the contact **attractive interaction**.

- In general, one introduces auxiliary **dimer fields** $\phi \simeq \psi_\uparrow \psi_\downarrow$ through a **Hubbard-Stratonovich transformation**

$$\mathcal{S}[\psi] = \int_x \left[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2M} - \mu \right) \psi_\sigma + \nu_d \phi^\dagger \phi + h(\phi^\dagger \psi_\uparrow \psi_\downarrow + \text{H.c.}) \right].$$



Ansatz for the effective action

- A common **truncated ansatz** for the **Bose gas** is

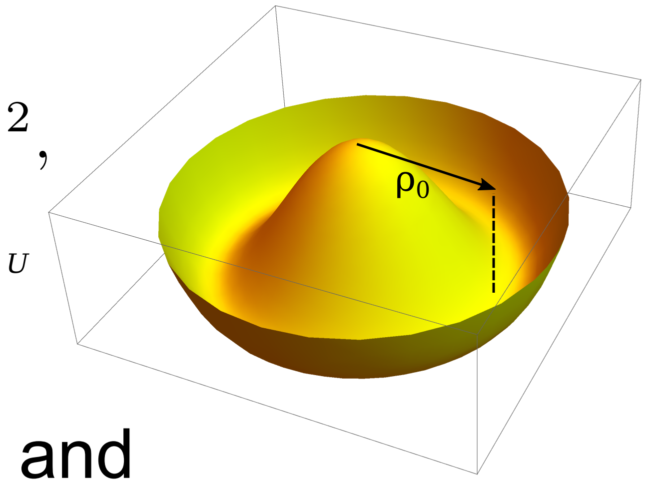
$$\Gamma_k = \int_x \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2M} - \mu\right)\psi_\sigma + \phi^\dagger \left(S_{\phi,k}\partial_\tau - \frac{Z_{\phi,k}}{4M}\nabla^2 + V_{\phi,k}\partial_\tau\right)\phi$$

$$+ U_k(\rho) - h\left(\phi^\dagger\psi_\uparrow\psi_\downarrow + \text{h.c.}\right)\Bigg],$$

where $\rho = \phi^\dagger\phi$, and

$$U_k(\rho) = u_{0,k} + m^2_{\phi,k}(\rho - \rho_{0,k}) + \frac{\lambda_{\phi,k}}{2}(\rho - \rho_{0,k})^2,$$

is the **effective potential**.



- The **couplings** $Z_{\phi,k}$, $S_{\phi,k}$, $V_{\phi,k}$, $u_{k,0}$, $m^2{}_{\phi,k}$, $\lambda_{\phi,k}$, and $\rho_{0,k}$ **depend on the scale** k. We keep h fixed.

Initial conditions

- The **initial conditions** are

$$m^2_{\phi,\Lambda} = \nu_\phi, \qquad \rho_{0,\Lambda} = 0, \qquad Z_{\phi,\Lambda} = S_{\phi,\Lambda} = V_{\phi,\Lambda} = 0.$$

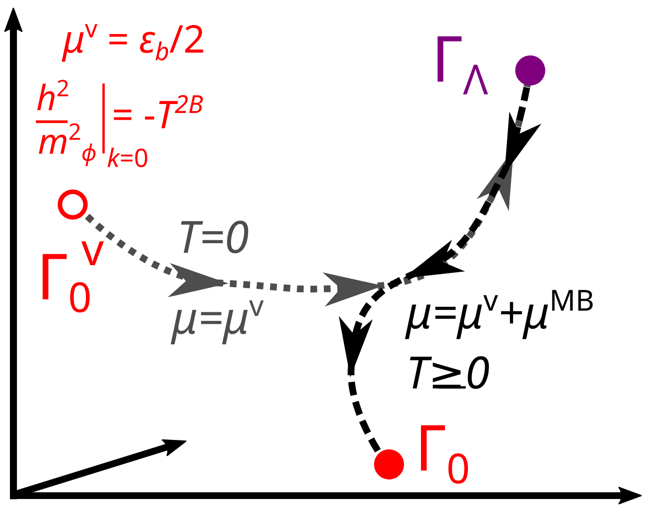
- Unlike the repulsive Bose gas, the **flow starts** in the **symmetric phase**.

- We impose that the flow in **vacuum** $(\mu{=}\mu^{\text{v}}, T{=}0)$ recovers the **known two-body physics**

$$\left.\frac{h^2}{m^2_{\phi,k}}\right|^{(\text{vacuum})}_{k\to 0} = -T^{(2\text{B})},$$

where in **three dimensions**

$$T^{(2\text{B})} = \frac{4\pi a}{M}, \qquad \underbrace{\epsilon_b = -\frac{1}{Ma^2}\Theta(a).}_{\text{Two-body binding energy}}$$



Regulator

- For the employed ansatz, we can use the **optimised Litim regulator** to solve the **momentum integrals analytically**.
- For the **bosonic sector**, it reads

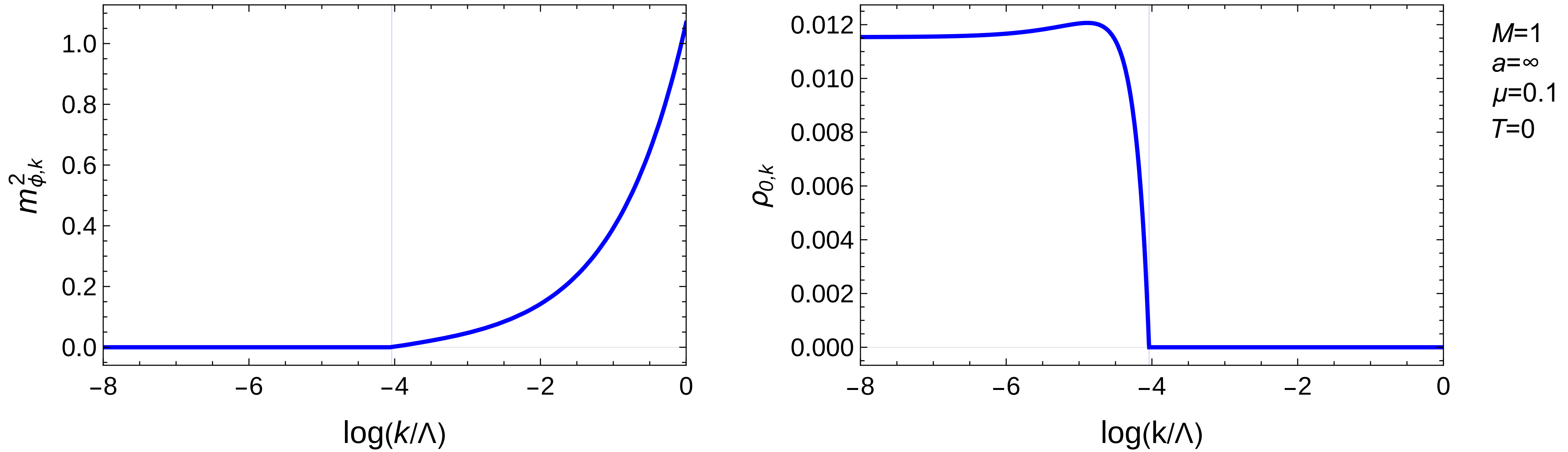
$$R_{B,k}(\boldsymbol{q}) = \frac{Z_{\phi,k}}{4M}(2k^2 - \boldsymbol{q}^2)\Theta(2k^2 - \boldsymbol{q}^2).$$

- For the **fermionic sector**, it reads

$$R_k(\boldsymbol{q}) = \frac{1}{2M}(k^2 \mathrm{sgn}(\boldsymbol{q}^2 - p_F^2) - (\boldsymbol{q}^2 - p_F^2))\Theta(k^2 - |\boldsymbol{q}^2 - p_F^2|),$$

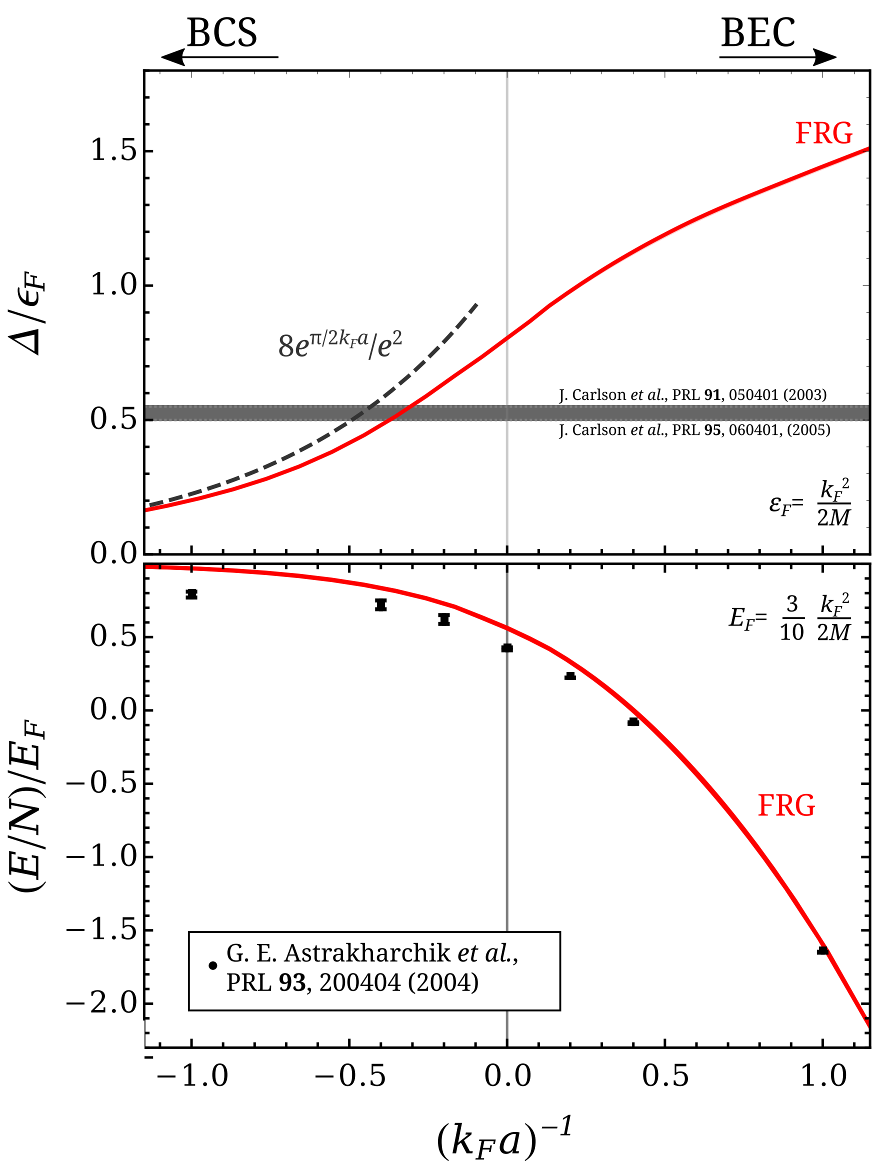
where $p_F^2 = -2M\mu$. This **regulates** the momentum **around the Fermi surface**.

Flow examples (d=3, T=0)



- The **FRG** works well in the **Unitary limit** $a \to \infty$.
- The **flow ends** in the **broken phase**, as expected for a zero-temperature Fermi gas.

Example (d=3, T=0)



- The **pairing gap** is obtained from

$$\Delta_k = h\rho_{0,k}^{1/2}.$$

- The thermodynamics are obtained as in the Bose gas.
- This simple ansatz can give a **good qualitative description**.
- Here we stress that the **BCS-BEC crossover** is **strongly-interacting**.
- Further truncations can provide an accurate description.

S. Floerchinger, M. M. Scherer, and C. Wetterich, Phys. Rev. A **81**, 063619 (2010).
I. Boettcher, J. M. Pawlowski, and C. Wetterich, Phys. Rev. A **89**, 053630 (2014).

Lecture 3

1. The **FRG** for **Bose gases**. Ansatz within the **derivative expansion**.
2. Calculation of **observables**.
3. The **FRG** for **Fermi gases**.
4. **Other applications** and overview.

General strategy

- In many cases, the **driving terms** can be computed **by hand**.
- However, in general, one generates them with a **symbolic computation language**.
- These can be done with public available libraries
  M. Q. Huber, A. K. Cyrol, J. M. Pawlowski, Phys. Commun. **248**, 107058 (2020).
  J. M. Pawlowski, C. S. Schneider, N. Wink, Comput. Phys. Commun. **287**, 108711 (2023).
- The **flow equations** can be solved with standard numerical routines for differential equations.
  - → Sometimes one needs to be careful with **stiff flows**.

Lattice systems

PHYSICAL REVIEW B **83**, 172501 (2011)

Nonperturbative renormalization group approach to the Bose-Hubbard model

A. Rançon and N. Dupuis
Laboratoire de Physique Théorique de la Matière Condensée, CNRS UMR 7600, Université Pierre et Marie Curie, 4 Place Jussieu, F-75252 Paris Cedex 05, France
(Received 18 February 2011; published 18 May 2011)

We present a nonperturbative renormalization group (RG) approach to the Bose-Hubbard model. By taking as initial condition of the RG flow the (local) limit of decoupled sites, we take into account both local and long-distance fluctuations in a nontrivial way. This approach yields a phase diagram in very good quantitative agreement with the quantum Monte Carlo results and reproduces the two universality classes of the superfluid–Mott-insulator transition with a good estimate of the critical exponents. Furthermore, it reveals the crucial role of the "Ginzburg length" as a crossover length between a weakly and a strongly correlated superfluid phase.

DOI: 10.1103/PhysRevB.83.172501 PACS number(s): 67.25.dj, 05.30.Jp, 05.10.Cc, 05.30.Rt

A. Rançon and N. Dupuis, Phys. Rev. B **83**, 172501 (2011).

PHYSICAL REVIEW B **84**, 174513 (2011)

Nonperturbative renormalization group approach to strongly correlated lattice boso

A. Rançon and N. Dupuis
Laboratoire de Physique Théorique de la Matière Condensée, CNRS UMR 7600 and Université Pierre et Marie Curie, 4 Place Jussieu, FR-75252 Paris Cedex 05, France
(Received 5 July 2011; revised manuscript received 10 October 2011; published 21 November 2011)

We present a nonperturbative renormalization-group approach to the Bose-Hubbard model. By taking as initial condition of the renormalization-group flow the (local) limit of decoupled sites, we take into account both local and long-distance fluctuations in a nontrivial way. This approach yields a phase diagram in very good quantitative agreement with quantum Monte Carlo simulations, and reproduces the two universality classes of the superfluid-Mott-insulator transition. The critical behavior near the multicritical points, where the transition takes place at constant density, agrees with the original predictions of Fisher *et al.* [Phys. Rev. B **40**, 546 (1989)] based on simple scaling arguments. At a generic transition point, the critical behavior is mean-field like with logarithmic corrections in two dimensions. In the weakly correlated superfluid phase (far away from the Mott insulating phase), the renormalization-group flow is controlled by the Bogoliubov fixed point down to a characteristic (Ginzburg) momentum scale k_G, which is much smaller than the inverse healing length k_h. In the vicinity of the multicritical points, when the density is commensurate, we identify a sharp crossover from a weakly to a strongly correlated superfluid phase where the condensate density and the superfluid stiffness are strongly suppressed and both k_G and k_h are of the order of the inverse lattice spacing.

A. Rançon and N. Dupuis, Phys. Rev. B **84**, 174513 (2011).

Mixtures and polarons

PHYSICAL REVIEW A **83**, 063620 (2011)

Excitation spectra and rf response near the polaron-to-molecule transition from the functional renormalization group

Richard Schmidt and Tilman Enss
Physik Department, Technische Universität München, D-85747 Garching, Germany

(Received 11 April 2011; published 15 June 2011)

A light impurity in a Fermi sea undergoes a transition from a polaron to a molecule for increasing interaction. We develop a method to compute the spectral functions of the polaron and molecule in a unified framework based on the functional renormalization group with full self-energy feedback. We discuss the energy spectra and decay widths of the attractive and repulsive polaron branches as well as the molecular bound state, and confirm the scaling of the excited-state decay rate near the transition. The quasiparticle weight of the polaron shifts from the attractive to the repulsive branch across the transition, while the molecular bound state has a very small residue characteristic for a composite particle. We propose an experimental procedure to measure the repulsive branch in a ^{6}Li Fermi gas using rf spectroscopy and calculate the corresponding spectra.

R. Schmidt and T. Enss, Phys. Rev. A **83**, 063620 (2011).

Renormalization-group study of Bose polarons

Felipe Isaule,[1] Ivan Morera,[1] Pietro Massignan,[2] and Bruno Juliá-Díaz[1]
[1]*Departament de Física Quàntica i Astrofísica, Facultat de Física, and Institut de Ciències del Cosmos, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain*
[2]*Departament de Física, Universitat Politècnica de Catalunya, Campus Nord B4-B5, E-08034 Barcelona, Spain*

(Received 30 May 2021; accepted 5 August 2021; published 19 August 2021)

We study the properties of a single impurity in a dilute Bose gas, a Bose polaron, using the functional renormalization group. We use an ansatz for the effective action motivated by a derivative expansion, and we compute the energies of the attractive and repulsive branches of excitations in both two and three spatial dimensions. Three-body correlations play an important role in the attractive branch, and we account for those by including three-body couplings between two bath bosons and the impurity. Our calculations compare very favorably with state-of-the-art experimental measurements and numerical simulations.

DOI: 10.1103/PhysRevA.104.023317

F. Isaule, I. Morera, P. Massignan, and B. Juliá-Díaz, Phys. Rev. A **104**, 023317 (2021).

PHYSICAL REVIEW A **105**, 013317 (2022)

Functional-renormalization-group approach to strongly coupled Bose-Fermi mixtures in two dimensions

Jonas von Milczewski, Félix Rose, and Richard Schmidt
Max-Planck-Institute of Quantum Optics, Hans-Kopfermann-Straße 1, 85748 Garching, Germany and Munich Center for Quantum Science and Technology (MCQST), Schellingstraße 4, 80799 Munich, Germany

(Received 7 May 2021; revised 1 December 2021; accepted 22 December 2021; published 24 January 2022)

We study theoretically the phase diagram of strongly coupled two-dimensional Bose-Fermi mixtures interacting with attractive short-range potentials as a function of the particle densities. We focus on the limit where the size of the bound state between a boson and a fermion is small compared to the average interboson separation and develop a functional-renormalization-group approach that accounts for the bound-state physics arising from the extended Fröhlich Hamiltonian. By including three-body correlations we are able to reproduce the polaron-to-molecule transition in two-dimensional Fermi gases in the extreme limit of vanishing boson density. We predict frequency- and momentum-resolved spectral functions and study the impact of three-body correlations on quasiparticle properties. At finite boson density, we find that when the bound-state energy exceeds the Fermi energy by a critical value, the fermions and bosons can form a fermionic composite with a well-defined Fermi surface. These composites constitute a Fermi sea of dressed Feshbach molecules in the case of ultracold atoms, while in the case of atomically thin semiconductors a trion liquid emerges. As the boson density is increased further, the effective energy gap of the composites decreases, leading to a transition into a strongly correlated phase where polarons are hybridized with molecular degrees of freedom. We highlight the universal connection between two-dimensional semiconductors and ultracold atoms, and we discuss perspectives for further exploring the rich structure of strongly coupled Bose-Fermi mixtures in these complementary systems.

DOI: 10.1103/PhysRevA.105.013317

J. von Milczewski, F. Rose, and R. Schmidt, Phys. Rev. A **105**, 013317 (2022).

Nuclear matter

PHYSICAL REVIEW C **91**, 035802 (2015)

From asymmetric nuclear matter to neutron stars: A functional renormalization group study

Matthias Drews and Wolfram Weise
Physik Department, Technische Universität München, D-85747 Garching, Germany
and ECT, Villa Tambosi, I-38123 Villazzano (Trento), Italy*
(Received 24 December 2014; revised manuscript received 22 February 2015; published 17 March 2015)

A previous study of nuclear matter in a chiral nucleon-meson model is extended to isospin-asymmetric matter. Fluctuations beyond mean-field approximation are treated in the framework of the functional renormalization group. The nuclear liquid-gas phase transition is investigated in detail as a function of the proton fraction in asymmetric matter. The equations of state at zero temperature of both symmetric nuclear matter and pure neutron matter are found to be in good agreement with realistic many-body computations. We also study the density dependence of the pion mass in the medium. The question of chiral symmetry restoration in neutron matter is addressed; we find a stabilization of the phase with spontaneously broken chiral symmetry once fluctuations are included. Finally, neutron-star matter including β equilibrium is discussed. The model satisfies the constraints imposed by the existence of two-solar mass neutron stars.

DOI: 10.1103/PhysRevC.91.035802 PACS number(s): 26.60.Kp, 21.65.−f

M. Drews and W. Weise, Phys. Rev. C **91**, 035802 (2015).

Progress in Particle and Nuclear Physics 93 (2017) 69–107

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

Functional renormalization group studies of nuclear and neutron matter

CrossMark

Matthias Drews [a], Wolfram Weise [a,b,*]

[a] Physik-Department, Technische Universität München, D-85748 Garching, Germany
[b] Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, USA

ARTICLE INFO

Article history:
Available online 9 November 2016

Keywords:
Chiral symmetry
Nuclear many-body problem
Functional renormalization group
Nuclear thermodynamics
Dense baryonic matter
Neutron stars

ABSTRACT

Functional renormalization group (FRG) methods applied to calculations of isospin-symmetric and asymmetric nuclear matter as well as neutron matter are reviewed. The approach is based on a chiral Lagrangian expressed in terms of nucleon and meson degrees of freedom as appropriate for the hadronic phase of QCD with spontaneously broken chiral symmetry. Fluctuations beyond mean-field approximation are treated solving Wetterich's FRG flow equations. Nuclear thermodynamics and the nuclear liquid–gas phase transition are investigated in detail, both in symmetric matter and as a function of the proton fraction in asymmetric matter. The equations of state at zero temperature of symmetric nuclear matter and pure neutron matter are found to be in good agreement with advanced ab-initio many-body computations. Contacts with perturbative many-body approaches (in-medium chiral perturbation theory) are discussed. As an interesting test case, the density dependence of the pion mass in the medium is investigated. The question of chiral symmetry restoration in nuclear and neutron matter is addressed. A stabilization of the phase with spontaneously broken chiral symmetry is found to persist up to high baryon densities once fluctuations beyond mean-field are included. Neutron star matter including beta equilibrium is discussed under the aspect of the constraints imposed by the existence of two-solar-mass neutron stars.

M. Drews and W. Weise, Prog. Part. Nucl. Phys. **93**, 69 (2017).

Overview

- We have examined the use of the **FRG** for **Bose** and **Fermi** gases.
- The **FRG** is a powerful tool for studying **strongly-correlated systems** and **phase transitions**.
- It beautifully incorporates the concepts of **scales** and **correlations**.
- However, it is generally limited to **two**- and **three-dimensional systems**.